A Novel Fuzzy Modified RAFSI Method and its Applications in Multi-Criteria Decision-Making Problems

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In real-life decision-making problems, the constraints may change from time to time. Change in certain decision elements can lead to the introduction of new alternatives or the removal of old alternatives to the existing decision, resulting in rank reversal. Rank reversal is the most significant problem that can't be ignored in multi-criteria decision-making (MCDM) methods. Ranking of alternatives through functional mapping of criterion subintervals into a single interval (RAFSI) method effectively removes the problem of rank reversal, but there are some limitations like standardized decision matrix is obtained by the assumption of supreme value as at least six times improved than the anti-supreme value, which is not always true. This paper aims to address those limitations by giving a modified form of the RAFSI (MRAFSI) method. As real-life problems are associated with uncertainty in the form of linguistic terms, a fuzzified form of the MRAFSI method has been given using triangular fuzzy numbers (TFNs) to deal with uncertainty. The effectiveness of the presented method is illustrated using a real-time case study to rank five stocks under the National Stock Exchange (NSE) for the year 2021 and is compared with other MCDM methods for validation. The supplier selection problem has been taken as an example to show the application of the Fuzzy Modified RAFSI (FMRAFSI) method.

Povzetek: Študija predstavlja Fuzzy Modified RAFSI (FMRAFSI) metodo za reševanje problemov večkriterijskega odločanja (MCDM), ki obvladuje negotovost z uporabo trikotnih mehkih števil in zmanjšuje problem obratnega razvrščanja.

1 Introduction

MCDM methods proved as a very important tool in solving most real-world problems. But one of the foremost significant problems that can't be ignored in most of the MCDM methods is rank reversal, the matter of unpredicted modification within the ranking of alternatives with the addition of the latest alternative or removal of an old alternative. MCDM methods are also prone to rank reversal when a problem is decomposed into multiple smaller problems keeping the standard weight and alternative scores unaltered [1]. The key explanation for rank reversal is the use of normalization, which changes with the addition or deletion of alternatives. This distorts the initial data and violates the 'Principle of Independence from Irrelevant Alternatives (PIIA). This is often true for any normalization [2]. Since differences in dimensional units of attributes can only be eliminated by normalization in most of the MADM approaches it becomes a vital part.

During the utilization of the Analytic Hierarchy Process (AHP), the matter of rank reversal was initially observed by Belton and Gear [3]. The identical was also noticed by Triantaphyllou and Mann [4] in AHP during the substitution of the worst alternative with an anti-ideal alternative. Saaty and Varga [5] presented that the matter of rank reversal can happen because of the occurrence of almost identical copies within the set of alternatives. They also opined that the addition of a new alternative can practically modify the previous preference order. Fedrizzi et al., [6] presented that the possibility of rank reversal rests on the distribution of criteria weights i.e., the entropy of the weight distribution. They established that the projected possibility of rank reversal rises with the weight's entropy. Further many authors noticed this problem in several MCDM methods because of the mutual correlations between the relevant and irrelevant alternatives, as a consequence of normalization [7]. Wang and Elhag [8] presented a technique to evade rank reversal in AHP by preserving the local significance of alternatives with the introduction of a new alternative. Mufazzal and Muzakkir [9] proposed a proximity index to minimize the rank reversal in MCDM problems. Sałabun et al., [10] developed a new MCDM method called the Characteristic Objects Method (COMET). They established that it's better than AHP concerning rank reversal.

De Farias Aires and Ferreira [11] introduced an approach targeting the identification of rank reversal during the normalization process in the TOPSIS method. Yang and Wu [12] introduced a novel R-VIKOR-based method to address rank reversal problems. Majumdar et al., [13] investigated a novel form of rank reversal specifically within the Analytic Hierarchy Process (AHP), identifying the aggregation method and criteria weight normalization as pivotal factors contributing to its occurrence. Similarly, Liu and Ma [14] delved into the causes of rank reversal within the ELECTRE II method, offering insights into its evaluation. Additionally, Tiwari and Kumar [15] presented a robust rank reversal technique for cloud service selection using the TOPSIS method with a Gaussian distribution. Yang et al., [16] an adapted approach to minimize rank reversal occurrences within the classic TOPSIS method. However, within the previous couple of years, a huge number of advanced MADM methods gave effective outcomes for resolving real-world problems [17]. But a maximum of those methods are not able to effectively remove the matter of rank reversal.

There are abundant applications of MCDM methods in real-life problems. Some of the applications consist of construction method selection for green building projects, portfolio selection, business and marketing, supplier selection, healthcare management, wastewater management, transportation problems, site selection for solar thermoelectric power plants, infectious waste disposal, industry development, flood detection criteria, social media analysis, supply chain network design, etc. In such cases, if rank reversal exists, and that too of higher order, a non-optimal alternative gets selected, thus resulting in a big concession.

Zizovic et al., [18] developed a new method referred to as Ranking of alternatives through functional mapping of criterion subintervals into a single interval (RAFSI), and its fuzzified form has been used for solving the selection problem in health organizations for COVID-19 virus pandemic [19], and for choosing a group of construction machines for enabling mobility [20]. Although this method successfully removes the problem of rank reversal, some modifications may be done to this method to make it better for solving real-life problems. This paper aims to work on the modifications that can be made to the RAFSI method. Also, since real-life problems are associated with uncertainty in the form of linguistic terms, the fuzzified form of the MRAFSI method has been given using triangular fuzzy numbers (TFNs) to deal with uncertainty persisting in the real world. To show the applicability of the presented method it has been applied to two important decision-making problems namely indices selection and supplier selection problems. For validation comprehensive analysis has been done with other well-known MCDM methods.

The rest of the paper is organized as follows. Section 2 discusses the RAFSI method and its shortcomings. Section 3 presents the mathematical formulation of the modified RAFSI method with the real case study as an application along with the comparative analysis. Section 4 presents the fuzzification of the MRAFSI method with application and comparison with the traditional fuzzy MCDM methods. Section 5 discusses the theoretical basis of the proposed approach and compares it with existing approaches for rank reversal, followed by sensitivity analysis in Section 6. At last section 7 concludes the paper.

1.1 Related work

Extensive research has been conducted in the field of rank reversal, resulting in a vast body of literature. To gain insights into this domain, we conducted a comprehensive review of relevant studies and categorized them based on the approach employed, the method utilized, and the limitations identified. The classification of these studies is presented in Table 1, offering a systematic overview of the diverse research framework surrounding the rank reversal problem.

Table 1: Literature review on rank reversal approaches

Year	Author	Method	Limitations
2023	Saluja et al. [21]	Proximity indexed value (PIV)	Struggles with a substantial prevalence of rank reversal.
2023	Tu and Wu [22]	AHP	Intransitive preference and the prioritization methods cause rank reversals in single pairwise comparison matrices.
2023	Dehshiri and Firoozabadi [23]	Wins in league (WIL)	Sensitive to small changes, limited handling of uncertainty.
2022	Yang et al. [16]	IE-TOPSIS	Relies on supplementary data, potentially unable to eliminate rank reversal.
2021	Tiwari and Kumar [15]	G-TOPSIS	Reliance on Gaussian distribution assumptions, subjective user priority influence.
2021	Kizielewicz et al. [24]	Characteristic Objects method (COMET)	Potential sensitivity to minor variations in input data, uncertainties in handling fuzzy data representations, and a lack of robustness in maintaining consistent rankings.
2020	Stevic et al. [17]	MARCOS	Complex implementation, limited generalizability,

			sensitive to parameter changes.
2020	Zizovic et al. [18]	RAFSI	Subjective criterion interval setting, reliance on an arbitrary superiority threshold, and the potential for identical rankings among different alternatives due to its assumptions on criteria types.

2 RAFSI method

In this section, the RAFSI method given by Zizovic et al., [18] is discussed. Given the initial decision matrix with weights of criteria estimated by any of the known methods, the RAFSI method has the subsequent stages.

1) The DM describes ideal (a_{Ij}) and anti-ideal (a_{Nj}) values for individual criteria.
2) Mapping of elements of the decision matrix into criteria intervals.
   - $C_j \in [a_{Nj}, a_{Ij}]$, where C_j belongs to *max* type criteria.
   - $C_j \in [a_{Ij}, a_{Nj}]$, where C_j belongs to *min* type criteria.

   Mapping of subintervals into criteria interval $[n_1, n_{2k}]$ by the formula-

   $$f_s(x) = \frac{n_1 - n_{2k}}{a_{Ij} - a_{Nj}} x + \frac{a_{Ij} n_1 - a_{Nj} n_{2k}}{a_{Ij} - a_{N_j}}$$

   It is supposed that the optimal value is six times improved than the non-optimal value i.e., $n_1 = 1$ and $n_{2k} = 6$. In this way, a standardized decision matrix is obtained.
   - for *max* type criteria if $a_{xj} > a_{Ij}$, then $f(a_{xj}) = f(a_{Ij})$
   - for *min* type criteria if $a_{xj} < a_{Ij}$, then $f(a_{xj}) = f(a_{Ij})$
3) Next, calculate arithmetic and harmonic mean of n_1, n_{2k}.

   $$A = \frac{(n_1 + n_{2k})}{2}, H = \frac{2}{\frac{1}{n_1} + \frac{1}{n_{2k}}}$$

4) Find a normalized decision matrix
   - for max type criteria $\hat{s}_{ij} = \frac{s_{ij}}{2A}$
   - for min type criteria $\hat{s}_{ij} = \frac{H}{2s_{ij}}$
5) Calculate criteria functions of alternatives V(A_i).

   $$V(A_i) = \omega_1 \hat{s}_{i1} + \omega_2 \hat{s}_{i2} + \ldots + \omega_n \hat{s}_{in}$$

   Finally, alternatives are ranked in descending order of V(A_i).

2.1 Limitations of RAFSI method

This section discusses the limitations of the existing RAFSI method.

1) In this method the DM's set the interval for each criterion by assumption without the use of any standard formula.
2) In this method for forming a standardized decision matrix, it is supposed that the optimal value is at least six times better than the non-optimal value, but it is not always true.
3) This method assumes that
   - for max type criteria if $a_{xj} > a_{Ij}$, then $f(a_{xj}) = f(a_{Ij})$
   - for min type criteria if $a_{xj} < a_{Ij}$, then $f(a_{xj}) = f(a_{Ij})$

   but this may lead to the same ranking of two different alternatives.

The following example illustrates it more efficiently.

Example: Consider the initial decision matrix given below and let the criteria sub-intervals be defined as-

$C_1 \in [2, 10]$, $C_2 \in [4, 8]$, $C_3 \in [0, 5]$

$$A = \begin{matrix} \\ A1 \\ A2 \\ A3 \\ A4 \\ \end{matrix} \begin{bmatrix} C1 & C2 & C3 \\ 12 & 6 & 1 \\ 10 & 6 & 1 \\ 5 & 7 & 4 \\ 8 & 5 & 3 \\ max & max & min \end{bmatrix}$$

thus, according to RAFSI method $f(12) = f(10)$ for alternative A_1, and other values being same for alternatives A_1 and A_2 we get same rank for alternative A_1 and A_2. But as it can be seen since criteria C_1 is of the maximum type so A_1 must be at a higher rank than A_2.

3 Modified RAFSI (MRAFSI) method

In this section, we have tried to overcome the shortcomings of the RAFSI method. The flow chart of the MRAFSI method is shown in Figure 1.

Let the initial decision matrix consists of m alternative A_1, A_2, …. A_m and n criteria C_1, C_2, …… C_n. Find the weights of criteria by any one of the known methods considering the relative importance between criteria such that $\sum_{i=1}^{n} w_i = 1$. The initial decision matrix is shown as follows.

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$$

The MRAFSI has the following steps-

Step.1. Find intervals for each criterion using the mean (μ) and standard deviation (σ) of the values of criteria for different alternatives as given in the decision matrix.

$$[\mu-2\times\sigma, \mu+2\times\sigma] = [n_1, n_2]$$

Step.2. Find the normalized decision matrix S = $[s_{ij}]_{m*n}$ by the use of the following formula-

$$s_{ij} = \frac{1}{1+e^{-x}} \tag{1}$$

here,

$$x = \frac{a_{ij}-n_1}{n_2-n_1} \text{ for beneficial criteria}$$

$$x = \frac{n_2-a_{ij}}{n_2-n_1} \text{ for non-beneficial criteria}$$

Step.3. Calculate the criteria functions of alternative $V(A_i)= w_1 s_{i1} + w_2 s_{i2} + \ldots + w_n s_{in}$ (2)

where $w_1, w_2, \ldots\ldots w_n$ represents the weight of criteria.

Finally, rank the alternatives in descending order of $V(A_i)$.

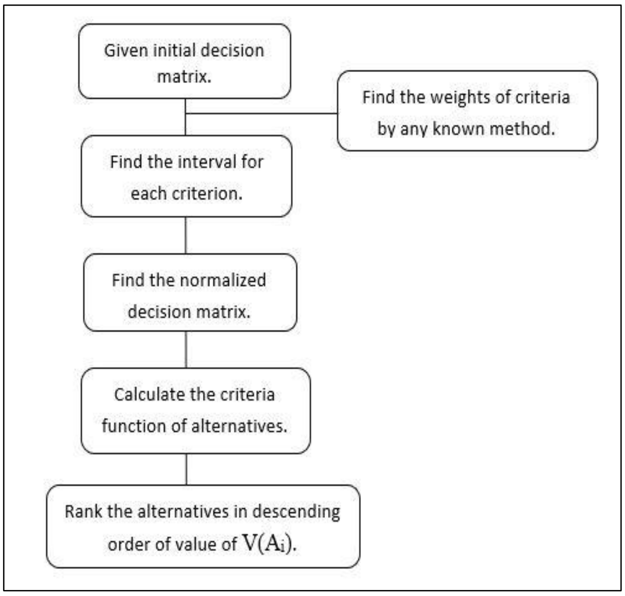

Figure 1: Block diagram of the MRAFSI method

3.1 Applications of MRAFSI multi-criteria model

This section presents the application of the MRAFSI methodology for the stock selection problem. A real case example of NSE (National Stock Exchange) is shown for selecting the best indices out of the given four indices Hindustan unilever (A_1), Asian paints (A_2), Tata consultancy services (A_3), Reliance industries (A_4) with four criteria Return on equity (ROE) (C_1), Earning per share (EPS) (C_2), Face value (C_3), P/E ratio(C_4) of year 2021 downloaded from www.ratestar.in. The weights of each criterion are given by w_i = (0.104445,0.13603,0.645511,0.114014) found by the entropy method. The decision matrix is demonstrated below.

$$\begin{array}{c} \\ A1 \\ A2 \\ A3 \\ A4 \\ \\ \end{array}\begin{bmatrix} C1 & C2 & C3 & C4 \\ 28.63 & 37.34 & 1 & 56.10 \\ 27.71 & 31.82 & 1 & 90.83 \\ 38.55 & 102.11 & 1 & 34.83 \\ 9.27 & 98.51 & 10 & 27.87 \\ max & max & max & min \end{bmatrix}$$

Applying the steps of MRAFSI method-

Step.1. Find the criteria subintervals using the mean and standard deviation of each column.

$C_1 \in [1.62,50.45]$; $C_2 \in [-8.6,143.53]$;
$C_3 \in [-5.75,12.25]$; $C_4 \in [-4.17,108.98]$;

Step.2. Find the normalized decision matrix by applying eq.1.

$$f_{A_1}(C_1) = \frac{1}{1+e^{\frac{-(28.63-1.62)}{(50.45-1.62)}}} = 0.634839$$

similarly solving other values, the normalized decision matrix can be obtained and as shown below:

$$\begin{array}{c} \\ A1 \\ A2 \\ A3 \\ A4 \\ \\ \end{array}\begin{bmatrix} C1 & C2 & C3 & C4 \\ 0.6348 & 0.5749 & 0.59267 & 0.6148 \\ 0.6305 & 0.5661 & 0.59267 & 0.5400 \\ 0.6805 & 0.6743 & 0.59267 & 0.6582 \\ 0.5391 & 0.6691 & 0.70578 & 0.7191 \\ max & max & max & min \end{bmatrix}$$

Step.3. Using eq. 2. find the criteria functions $V(A_i)$ of alternatives and rank them in descending order of $V(A_i)$ as shown in Table 2 and Figure 2.

Table 2: Final ranking of alternatives

Alternatives	$V(A_i)$	Rank
Hindustan unilever	0.597184	3
Asian Paints	0.586997	4
Tata consultancy services	0.620423	2
Reliance industries	0.679521	1

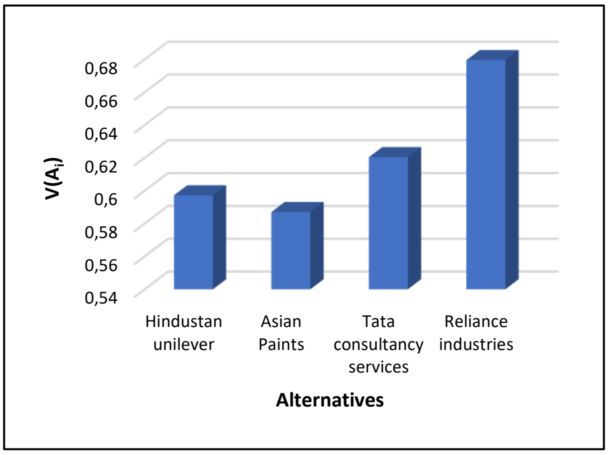

Figure 2: Ranking of stocks

Based on the above results, we found that Reliance industries is the best stock to invest in.

3.2 Rank reversal problem

The four alternatives are ranked according to MRAFSI method, now we need to check rank if we remove one alternative from them. Let us remove the alternative Hindustan unilever from the given alternatives. We find that the on removing the alternative of rank 3rd all the alternatives, after that alternative shift one rank up, without causing any rank reversal. Thus, it is observed that MRAFSI method gives effective results in dynamic environment as shown in Table 3.

Table 3: Ranking after removing one alternative

Alternatives	**V(Ai)**	**Rank**
Asian Paints	0.586997	3
Tata consultancy services	0.620423	2
Reliance industries	0.679521	1

Now let us add another alternative tata steel to the given four alternatives and check the rank. The new decision matrix formed is given below.

$$\begin{matrix} \\ A1 \\ A2 \\ A3 \\ A4 \\ A5 \\ \end{matrix}\begin{bmatrix} C1 & C2 & C3 & C4 \\ 28.63 & 37.34 & 1 & 56.10 \\ 27.71 & 31.82 & 1 & 90.83 \\ 38.55 & 102.11 & 1 & 34.83 \\ 9.27 & 98.51 & 10 & 27.87 \\ 10.87 & 317.21 & 10 & 4.3 \\ max & max & max & min \end{bmatrix}$$

After applying the steps of the MRAFSI method we found the rank of alternatives as shown below in Table 4.

Table 4: Ranking after adding one alternative

Alternatives	**V(Ai)**	**Rank**
Hindustan Unilever	0.591028	4
Asian Paints	0.575865	5
Tata consultancy services	0.613342	3
Reliance industries	0.6429	2
Tata steel	0.681706	1

The added alternative stood first in the ranking order, so all the alternatives moved single place down in the order. Thus, the MRAFSI method is resistant to rank reversal problems on adding and removing new alternatives.

3.3 Comparative analysis

For validation, the results obtained by MRAFSI method is compared with other known traditional MCDM methods. The same weights and initial decision matrix are taken in all other methods for comparison of the performance. Table 5 shows the ranking of alternatives using different methods.

Table 5: Ranking obtained by different methods

Method	**Ranking**	**Best alternative**	**Worst alternative**
MRAFSI	A4>A3>A1>A2	A4	A2
TOPSIS	A4>A3>A1>A2	A4	A2
COPRAS	A4>A3>A1>A2	A4	A2
MAUT	A4>A3>A1>A2	A4	A2

It is clear from the above table that there is no conflict in the ranking order of best and worst alternatives by all methods. Hence, this validates the MRAFSI method.

4 Fuzzy MRAFSI method

In this section, we present the fuzzified form of the MRAFSI method. This helps in handling the uncertainty persisting in real-life problems. Fuzzification is performed by applying triangular fuzzy numbers A= (a_1, a_2, a_3), where a_1 presents the smallest likely value, a_2 presents the most probable value and a_3 presents the largest possible value of any fuzzy event. Triangular fuzzy numbers (TFNs), being a specialized case of generalized fuzzy numbers, offer a competent way to present ambiguous information and linguistic preferences. The easy properties of TFNs captivated our attention to design the fuzzy RAFSI method to process the ambiguous information in the form of TFNs.
The fuzzy MRAFSI has the following stages-

Step.1. Formation of the fuzzy initial decision matrix. This matrix is formed by evaluating m alternatives (A_1, A_2, …. A_m) on *n* criteria C_1, C_2, …… C_n. The decision matrix is shown below.

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$$

where $a_{ij} = (a_{ij}^l, a_{ij}^s, a_{ij}^u)$ denotes the triangular fuzzy number.

Step.2. Find the criteria interval, by finding the mean and standard deviation for each element of TFNs. After finding the ideal and anti-ideal value in form of TFN we have the fuzzy criteria interval.

$C_j \in [n_1, n_2] \quad j = 1,2,3 \ldots n$

where n_1 and n_2 are TFN's.

Step.3. Convert the initial decision matrix into normalized matrix S = $[s_{ij}]_{m*n}$ by applying the formula

$$s_{ij} = \frac{(1,1,1)}{(1,1,1)+e^{-x}} \tag{3}$$

here,

$$x = \frac{a_{ij}-n_1}{n_2-n_1} \text{ for beneficial criteria}$$

$$x = \frac{n_2-a_{ij}}{n_2-n_1} \text{ for non-beneficial criteria}$$

a_{ij}, n_1, n_2 are all TFN's.
For solving equation (3) use the operations of triangular fuzzy numbers.

Step.4. Calculate the fuzzy criteria functions of alternatives $V(A_i)$ by applying the expression:

$$V(A_i)=w_1 s_{i1} + w_2 s_{i2} + \ldots + w_n s_{in} \quad (4)$$

where w_j represents the weights of criteria, which an be found by applying any of the known methods of weight determination. Here weight determination is not taken into consideration, they are assumed to be already known.

Step.5. Defuzzification of the fuzzy criteria functions of alternatives $V(A_i)$ is done by applying the expression:

$$V(A_i) = \frac{[V(A_i)^l + 4 * V(A_i)^s + V(A_i)^u]}{6} \quad (5)$$

Now rank the alternatives in the descending order of value of $V^*(A_i)$.

4.1 Applications of Fuzzy MRAFSI multi-criteria model

This section presents application of Fuzzy MRAFSI method for the supplier selection problem. An automobile company desires to select raw material suppliers. Three suppliers (S_1, S_2, S_3) are to be selected based on five criteria:

1. Quality supplied item (C_1)
2. Cost of supplied item (C_2)
3. Delivery time of supplied item (C_3)
4. Technology of supplied item (C_4)
5. Flexibility of supplied item (C_5)

The linguistic variables for weights are shown in Table 6.

Table 6: Linguistic variables for weights

Linguistic Variables	Ratings
Very Low (VL)	(0,0.1,0.2)
Low (L)	(0.1,0.3,0.5)
Medium (M)	(0.3,0.5,0.7)
High (H)	(0.6,0.8,0.9)
Very High (VH)	(0.8,0.9,1.0)

Weights of the criteria are given as:

w_1 = (0.83,0.97,1)
w_2 = (0.63,0.83,0.97)
w_3 = (0.77,0.93,1)
w_4 = (0.57,0.77,0.93)
w_5 = (0.5,0.7,0.9)

Applying the steps of fuzzy MRAFSI method to the given problem.

Step.1. Form the Fuzzy decision matrix using linguistic variables for rating shown in Table 7.

Table 7: Linguistic variables for rating

Linguistic Variables	Ratings
Very Poor (VP)	(0,1,2)
Poor (P)	(1,3,5)
Medium (M)	(3,5,7)
Good (G)	(6,8,9)
Very Good (VG)	(8,9,10)

The fuzzy decision matrix is shown below in Table 8 for the given problem.

Table 8: Fuzzy decision matrix

	C1	**C2**	**C3**	**C4**	**C5**
S1	(8.33,9.67,10)	(7.67,9.33,10)	(7.67,9.33,10)	(7,9,10)	(7,9,10)
S2	(5.67,7.6,9.3)	(3.67,5.67,7.6)	(3.67,5.67,7.6)	(3.67,5.67,7.6)	(4.33,6.33,8.3)
S3	(7,8.67,9.67)	(4.33,6.33,8.3)	(4.33,6.33,8)	(5.67,7.67,9.3)	(1.67,3.67,5.6)
	max	min	min	max	max

Step.2. Find the criteria interval by taking the mean and standard deviation of each element of TFN's in the criteria column as shown in Table 9.

Table 9: Interval for first criteria

	8.33	9.67	10
	5.67	7.67	9.33
	7	8.67	9.67
Mean(μ)	7	8.67	9.67
S. D (σ)	1.08	0.82	0.27
μ-2* σ	4.84	7.03	9.13
μ+2* σ	9.16	10.31	10.21

Thus, the interval for C_1 becomes:
$C_1 \in$ [(4.84,7.03,9.13), (9.16,10.31,10.21)]
Similarly, we find intervals for all other criteria:
$C_2 \in$ [(1.72,3.92,6.7), (8.72,10.3,10.63)]
$C_3 \in$ [(1.72,3.92,6.5), (8.72,10.3,10.62)]
$C_4 \in$ [(2.7,4.7,7.04), (8.18,10.18,10.95)]
$C_5 \in$ [(0,1.98,4.43), (8.68,10.68,11.56)]

Step.3. Find the normalized matrix by applying equation (3).

$$f_{A_1}(C_2) = \frac{(1,1,1)}{(1,1,1)+e^{\frac{-((8.72,10.3,10.63)-(7.67,9.33,10))}{((8.72,10.3,10.63)-(1.72,3.92,6.7))}}}$$

$$= \frac{(1,1,1)}{(1,1,1)+e^{-(-0.63,0.15,0.146)}} = \frac{(1,1,1)}{(2.88,1.86,1.23)}$$

$$= (0.35,0.54,0.81)$$

Similarly solving other values, we get the normalized matrix as shown in Table 10.

Table 10: Normalized decision matrix

	C1	**C2**	**C3**	**C4**	**C5**
S1	(0,0.69,1)	(0.35,0.54,0.81)	(0.36,0.54,0.79)	(0.49,0.69,0.98)	(0.55,0.69,0.84)
S2	(0,0.55,1)	(0.53,0.67,0.97)	(0.53,0.67,0.96)	(0.05,0.54,0.7)	(0.49,0.62,0.73)
S3	(0,0.62,1)	(0.51,0.65,0.96)	(0.52,0.65,0.94)	(0.23,0.63,0.93)	(0.34,0.55,0.6)
	max	min	min	max	max

Step.4. Using eq. (4) calculate the final fuzzy criteria functions of alternatives $V(A_i)$.

Step.5. Final ranking of alternatives is done after defuzzification of fuzzy criteria functions of alternatives $V^*(A_i)$, as shown in Table 11 and Figure 3.

Table 11: Ranking of alternatives

Alternative	$V(A_i)$	$V^*(A_i)$	Ranking
S_1	(1.05,2.63,4.24)	2.635	1
S_2	(1.01,2.57,4.21)	2.585	3
S_3	(1.03,2.62,4.28)	2.630	2

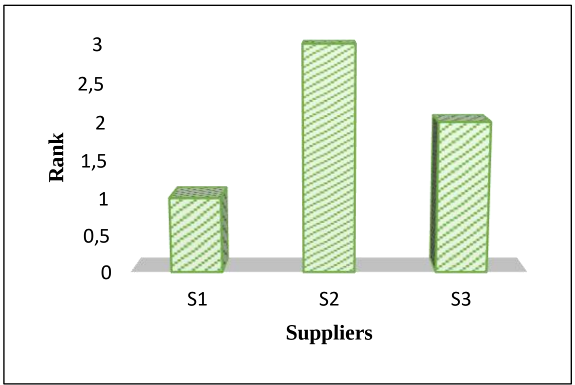

Figure 3: Ranking of suppliers

Based on the above results, we found that supplier 1 is the best alternative.

4.2 Comparative analysis

For validation, the results obtained by the FMRAFSI method is compared with the well-known Fuzzy TOPSIS and Fuzzy VIKOR method. The same weights and initial decision matrix are taken for comparison of the performance. Table 12 shows the ranking of alternatives using different methods.

Table 12: Comparison of ranking order

Method	Ranking	Best alternative	Worst alternative
FMRAFSI	A1>A3>A2	A1	A2
FTOPSIS	A1>A3>A2	A1	A2
FVIKOR	A1>A3>A2	A1	A2
FCOPRAS	A1>A3>A2	A1	A2
FELECTRE	A1>A3>A2	A1	A2
FPROMETHE	A1>A2>A3	A1	A3

It is clear from the above table that there is no conflict in the ranking order of best alternatives by different methods. Hence, this validates the FMRAFSI method.

5 Discussions

5.1 Theoretical basis

The rationale behind the mathematical formulation of mean and standard deviation in the modified RAFSI method is explained below:

Simplicity: This method offers a straightforward and easy-to-understand approach to estimate the mean and standard deviation of TFNs. By breaking down the TFN into its three values (lower, middle, upper), it simplifies the calculation process.

Transparency: It provides a transparent representation of the TFN's uncertainty. By using arithmetic operations (e.g., mean calculation, standard deviation computation) on individual terms, it offers an intuitive way to understand how these terms contribute to the overall statistics of the TFN.

Computational efficiency: Compared to some more complex methods like Monte Carlo simulation or PDF-based approaches, this method is computationally efficient. It avoids the need for extensive simulations or intricate mathematical formulations, making it suitable for quick estimations.

Applicability: This method might be particularly useful in scenarios where simplicity and a quick estimation of the mean and standard deviation are required. It can serve as a preliminary or initial estimation method, especially when dealing with a large number of TFNs in decision-making or uncertainty analysis contexts.

5.2 Comparative analysis

This section conducts a comparative analysis between the proposed approach and other methodologies for addressing rank reversal, as outlined in Table 1. It aims to elucidate the advantages inherent in the proposed approach when compared with existing methods.

1. **Stability against rank reversals:** Unlike methods such as Proximity Indexed Value (PIV), AHP, Wins in league (WIL), IE-TOPSIS, G-TOPSIS, and others prone to rank reversals, the Modified RAFSI method is designed to potentially mitigate the prevalence of rank reversals. It aims to produce more stable and consistent rankings, enhancing the reliability of decision-making processes.
2. **Enhanced handling of uncertainty:** Compared to methods like the Characteristic Objects method (COMET), which struggle with uncertainties and fuzzy data representations, Modified RAFSI offers improved handling of uncertainty. It provides a more robust means of dealing with fuzzy data representations, resulting in more reliable and consistent rankings even in uncertain scenarios.

3. **Reduced sensitivity to small changes:** In contrast to methods sensitive to small changes, such as Wins in league (WIL) and others, Modified RAFSI demonstrates lower sensitivity to minor fluctuations or variations in input data. This characteristic leads to more stable and robust rankings, less likely to be affected by insignificant changes.
4. **Objective ranking:** Similar to G-TOPSIS, RAFSI minimizes subjective bias. It aims to provide a more objective approach, enhancing the credibility and reliability of the rankings by minimizing the influence of subjective user assumptions.
5. **Simplicity and Generalizability:** Unlike complex methods like MARCOS, Modified RAFSI offers a more straightforward implementation while maintaining robustness and applicability across diverse decision-making scenarios. Its simplicity does not compromise its effectiveness in producing meaningful and reliable rankings.
6. **Reduced reliance on supplementary data:** RAFSI's design aims to reduce dependency on supplementary data, similar to how it is with IE-TOPSIS. This characteristic contributes to its practicality and efficiency, allowing it to generate rankings without relying heavily on additional information.

6 Sensitivity analysis

Decision-making is a multifaceted process susceptible to various potential errors. Therefore, a comprehensive analysis before model adoption becomes imperative. This typically involves conducting a sensitivity analysis, which can be executed through diverse approaches such as altering weight coefficients of criteria, changing measurement units expressing alternative values, comparing with alternate methodologies, etc. [25]. Most authors commonly perform sensitivity analyses focusing on adjustments in weight coefficients of criteria [26-27], as is the case in this paper as well. The primary objective of this sensitivity analysis is to gauge the impact of the most influential criterion on the ranking performance of the proposed model [28]. For the sensitivity analysis involving changes in weight coefficients, five distinct scenarios are developed. The basis for the change in weight coefficients makes the change in the weight coefficient of the best criterion C3. The changes in the weight coefficients of this criterion are made in interval $w_3 \in [0, 0.5]$.

The proportion set in this way always provides the condition where $\sum_{i=1}^{4} w_i = 1$. The values of the weight coefficients in all scenarios are shown in Figure 4.

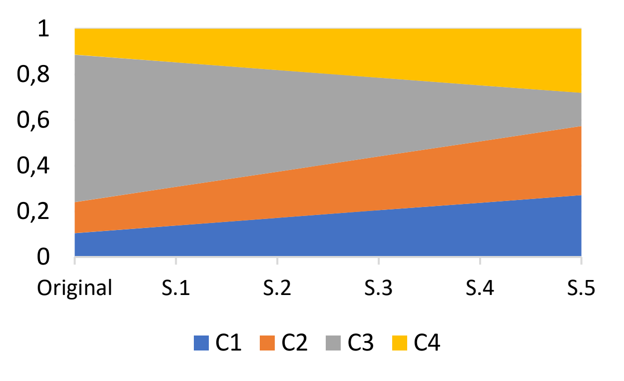

Figure 4: Weights under different scenarios

To further verify the stability of the proposed approach to attribute weights obtained by different methods, we use the objective weights obtained by critic and standard deviation method in place of weights obtained by entropy weights in the example. The weights obtained by different methods are shown in Table 13.

Table 13: The weight vector by different methods

Methods	w_1	w_2	w_3	w_4
Entropy	0.10444	0.13603	0.64551	0.11401
Critic	0.36515	0.18964	0.28223	0.16296
St. dev.	0.2186	0.28373	0.26211	0.23555

The ranking of alternatives by different scenarios and weight determination methods is shown in Table 14. It can be easily observed from Table 14 that although the weights differ greatly, a very small change in ranking results is seen. Thus, the proposed approach is stable in terms of ranking. To further verify the results the SSCs between the ranking obtained is calculated. From Table 15 it is observed that the SSCs between the ranking is greater than 0.8 under different weights. Thus, the proposed approach is stable under different weights.

Table 14: Ranking of alternatives by different scenarios

Alternative	**Original**	**Critic**	**St. Dev.**	**S1**	**S2**	**S3**	**S4**	**S5**
Hindustan unilever	3	3	3	3	3	3	3	3
Asian Paints	4	4	4	4	4	4	4	4
TCS	2	1	2	2	2	2	2	1
Reliance industries	1	2	1	1	1	1	1	2

Table 15: The SSCs between the ranking results

	Original	Critic	St. Dev.	S1	S2	S3	S4	S5
Original	1	0.8	1	1	1	1	1	0.8
Critic	-	1	0.8	0.8	0.8	0.8	0.8	1
St. Dev.	-	-	1	1	1	1	1	0.8
S1	-	-	-	1	1	1	1	0.8
S2	-	-	-	-	1	1	1	0.8
S3	-	-	-	-	-	1	1	0.8
S4	-	-	-	-	-	-	1	0.8
S5	-	-	-	-	-	-	-	1

7 Conclusions

This paper discusses the limitations of the RAFSI method and endeavors to address these deficiencies by introducing a modified RAFSI method (MRAFSI). To assess the efficacy of the proposed method, a real case study is conducted to rank five indices of the Bombay Stock Exchange (BSE) for the fiscal year 2020-21. Comparative analysis with established MCDM methods is performed to validate the modified approach, confirming the consistency in results and affirming the validity of the modified method.

In recognition of uncertainties prevalent in real-world scenarios, the MRAFSI method undergoes fuzzification using the triangular fuzzy numbers. The fuzzy modified RAFSI (FMRAFSI) is applied to a supplier selection problem. Comparative validation with traditional fuzzy methods is conducted, revealing congruent outcomes and thus affirming the validity of the FMRAFSI method. Additionally, a sensitivity analysis is carried out to showcase the resilience and reliability of the proposed approach.

For the future work, the proposed framework can be integrated to leverage hybrid models [29-30], thereby achieving more effective outcomes. It would be fascinating to use the proposed method to address a variety of further real-world decision-making issues.

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