Argumentation based machine learning for inconsistent knowledge bases

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Abstract. Knowledge integration in distributed data mining has received widespread attention that aims to integrate inconsistent information locating on distributed sites. Traditional integration methods become ineffective since they are unable to generate global knowledge, support advanced integration strategy, or make prediction without individual classifiers. In this paper, we propose an argumentation based reinforcement learning method to handle this problem. To this end, a constructive model to merge possiblistic belief bases built based on the famous general argumentation framework is proposed. An axiomatic model, including a set of rational and intuitive postulates to characterize the merging result is introduced and several logical properties are mentioned and discussed.

Keywords: Belief merging, Machine learning, Argumentation, Distributed data mining

1 Introduction

With the fast development and pervasive applications of data acquisition technologies, we have gain much more information than before and entered the era of big data. Traditional data mining techniques cannot handle the challenges of volume, velocity, variety and veracity in big data. As an alternative, distributed data mining (DDM) has received widespread attention recently, which aims to use distributed computing technology to extract knowledge from distributed data sources. With the advantages of better security and powerful computing capability, DDM can deal with data mining tasks on large-scale datasets. However, there are still some challenging problems in DDM, such as fragmentation, communication cost, integration and data skewness, in which the knowledge integration problem is currently a hotspot. In the first approach, we will try to adopt the inconsistency in obtaining information source by improving the classical reasoning methods. One of typical instances of this idea is the family of paraconsistent logics [16, 9, 8]. This approach needs a simple operation to collect and store information from source, but it requires a highly computational complexity reasoning operation. Unfortunately, the reasoning operation is more frequently used than another, thus this approach is only suitable for a specific class of applications.

In the second approach is using belief merging in which we try to build a consistent information system from multiple information sources. Precisely, from the given belief bases $\{K_1, \ldots, K_n\}$ we build a consistent belief base K^* which best represents for these belief bases.There are two settings in this approach, the centralized and distributed ones. In centralized setting, belief merging is considered as an arbitration in which all belief bases are submitted to a mediator, and this mediator will decide which is the common belief base. This is the main trend in belief merging with a large range of works such as [18–20, 23, 24]. Obviously, in this setting the merging result is depend on the mediator, the participating agents have to expose all their own beliefs and they are omitted in merging process. Therefore, it is difficult to apply to high interactive systems such as multiagent systems.

In the second setting, each merging process is considered as a game in which participants step by step give their proposals until an agreement is reached. The first direction in this setting is belief merging by negotiation with some typical works are as follows: a family of game-based merging operators[17], a two-stage belief merging process [10, 11], a bargaining game solution [30] and a game model for merging stratified belief bases [22, 25, 26, 28, 29].

The second direction is belief merging by argumentation in which merging process is organized as a debate and participants uses their own beliefs and manipulates argumentation skills to reach the agreement. Typically, an argumentation framework for merging weighted belief bases [15] and other framework for merging the belief bases in possibilistic logic by Amgoud et al. [3]. In [27], a general framework for merging belief bases by argumentation is introduced, however, the semantics of argumentation extensions are not mentioned and discussed.

In this work, we propose a new argumentation framework for merging possibilistic logic bases. The contribution of this paper is two-fold. First, we introduce a framework to merge possibilistic belief bases in which a general argumentation framework is applied in possibilistic belief bases to obtain meaningful results in comparison to other belief merging techniques for belief merging in possibilistic logic such as [3, 5, 4, 7, 21]. Second, an axiomatic model including rational and intuitive postulates for merging results is introduced and several logical properties are discussed.

The rest of this paper is organized as follows: We review about possibilistic logic in *Section II.* Belief merging for prioritized belief bases by possibilistic logic framework is presented in *Section III. Section IV* and *Section V* introduce a general argumentation framework and a model to merge belief bases by this framework. Postulates for belief merging by argumentation and logical properties is introduced an discussed in *Section VI.* Some conclusions and future work are presented in *Section VII.*

For the sake of representation, we consider the following example:

Example 1. A terrible environmental crisis, which cause mass fish deaths (a), happened in the seabed in Middle of Vietnam. There are some opinions ordered in time series as follows:
The public and scientists: The mass fish deaths (a) are caused by the toxic spill disaster of a steel factory (b): ($b \to a$).
Steel factory: We have a modern waste water treatment system (c), thus the water was cleaned before it was discharged: ($c, c \to \neg b$).
Communication agencies: The steel factory has imported hundreds of tons of chemical toxic (d) and its underground tube is put at wrong position (f): (d, f).
The public and scientists: A diver has died (g) with the symptom caused by toxin from water (b): ($g, b \to g$).
Steel factory: We have imported chemical material to detergent our tubes (d) however the water was cleaned before it was discharged: ($\neg(d \to b)$). Our underground tube is in the right position and it has not been completed, thus it can discharge now: ($\neg f$).

Official Authorities: There are two causes of mass fish deaths. It may be from chemical toxin (d) or it may cause by algae bloom phenomenon (e): $(d \rightarrow a) \vee (e \rightarrow a)$. We have not yet had any clue about the relation between mass fish deaths and the discharge of steel factory.
The public and scientists: The mass fish deaths cannot cause by the algae bloom phenomenon because there is no body of algae, water did not change color and fishes died at the bottom: $(\neg e)$.
From the progression of events as above, we have the sets of beliefs as follows:

$K_1 : \{b \rightarrow a, g, b \rightarrow g, \neg e\}$,
$K_2 : \{c, c \rightarrow \neg b, \neg(d \rightarrow b), \neg f\}$,
$K_3 : \{d, f\}$,
$K_4 : \{(d \rightarrow a) \vee (e \rightarrow a)\}$.

2 Possibilistic logic

In this work, we consider a propositional language $\mathcal{L}$ built on a finite alphabet $\mathcal{P}$ and common logic connectives including $\neg, \wedge, \vee$, and $\rightarrow$. The classical consequence relation is $\vdash$. We use Ω to denote a finite set of interpretations of $\mathcal{L}$. Given $\omega \in \Omega$, $\omega \models \psi$ represents that ω is a model of the formula ψ.
A *possibilistic formula* (ψ, α) includes a propositional formula ψ and a weight $\alpha \in [0, 1]$.
A *possibilistic belief base* is a finite set of possibilistic formulas $K = (\psi_i, \alpha_i)|i = 1, \ldots, n$.
We denote K^* an associated belief base w.r.t K defined as follows: $K^* = \{\psi_i|(\psi_i, \alpha_i) \in K\}$.
Obviously, a possibilistic belief base K is consistent if K^* is consistent and vice verse .
We also denote $\mathbb{K}$ and $\mathbb{K}^*$ set of all possibilistic belief bases and their associated belief bases, respectively.

For each possibilistic belief base K, the possibility distribution of K, denoted by π_K as follows:

Definition 1. *[13]* $\forall \omega \in \Omega$

$$\pi_K(\omega) = \begin{cases} 1 & \textit{if } \forall(\psi_i, \alpha_i) \in K, \omega \models \psi_i \\ 1 - max\{\alpha_i : (\psi_i, \alpha_i) \in K \textit{ and } \omega \not\models \psi_i\} & \textit{otherwise} \end{cases} \quad (1)$$

Example 2. Continuing Example 1.
Suppose that $K = \{(a, 0.8), (\neg c; 0.7), (b \rightarrow a, 0.6), (c; 0.5), (c \rightarrow \neg b; 0.4)\}$.
According to Definition 1, we can determine the possibility distribution for K *as follows:* $\pi_K(a\neg b\neg c) = 1, \pi_K(abc) = 0.6, \pi_K(ab\neg c) = 0.5, \pi_K(a\neg bc) = 0.3$, *and* $\pi_K(\neg abc) = \pi_K(\neg ab\neg c) = \pi_K(\neg a\neg bc) = \pi_K(\neg a\neg b\neg c) = 0.2$

Definition 2. *Given a possibilistic belief base* K *and* $\alpha \in [0, 1]$, *the* $\alpha - cut$ *of* K *is denoted by* $K_{\geq\alpha}$ *and defined as follows:* $(K_{\geq\alpha} = \{\psi \in K^*|(\psi, \beta) \in K, \beta \geq \alpha\})$. *Similarly, a* strict $\alpha - cut$ *of* K *is denoted by* $K_{>\alpha}$ *and defined as follows:* $(K_{>\alpha} = \{\psi \in K^*|(\psi, \beta) \in K, \beta > \alpha\})$.

Definition 3. *Possibilistic belief base* K_1 *is equivalent to possibilistic belief base* K_2, *written as* $K_1 \equiv K_2$ *if and only if* $\pi_{K_1} = \pi_{K_2}$.

It is easy to prove that $K_1 \equiv K_2$ iff for all $\alpha \in [0, 1]$ $(K_1)_{\geq\alpha} \equiv (K_2)_{\geq\alpha})$

2.1 Possibilistic inference

Definition 4. *The* inconsistency degree *of possibilistic belief base K is as follows:*

$$Inc(K) = max\{\alpha_i : K_{\geq \alpha_i} is\ inconsistent\} \tag{2}$$

The inconsistency degree of possibilistic belief base K is the maximal value α_i such that the $\alpha_i - cut$ of K is inconsistent. Conventionaly, if K is consistent, then $Inc(K) = 0$.

Definition 5. *Given a possibilistic belief base K and $(\psi, \alpha) \in K$, (ψ, α) is a* subsumption *in K if:*

$$(K \setminus \{(\psi, \alpha)\})_{\geq \alpha} \vdash \psi \tag{3}$$

Respectively, (ψ, α) is a strict subsumption in K if $K_{>\alpha} \vdash \psi$.

We have the following lemma [6]:

Lemma 1. *If (ψ, α) is a subsumption in K then $K \equiv (K \setminus \{(\psi, \alpha)\})$.*

Definition 6. *Given a possibilistic belief base K, formula ψ is a* plausible consequence *of K if:*

$$K_{>Inc(K)} \vdash \psi \tag{4}$$

Definition 7. *Given a possibilistic belief base K, formula (ψ, α) is a* possibilistic consequence *of K, denoted $K \vdash_\pi (\psi, \alpha)$, if:*

- $K_{>Inc(K)} \vdash \psi$
- $\alpha > Inc(K)$ *and* $\forall \beta > \alpha, K_{>\beta} \nvdash \psi$

In any inconsistent possibilistic belief base K, all formulas with certainty degrees smaller than or equal to $Inc(K)$ will be omitted in the inference process.

Example 3. Continuing Example 2, obviously K is equivalent to
$K' = \{(a, 0.8), (\neg c, 0.7), (b \rightarrow a, 0.6), (c, 0.5)\}$.
Formula $(c \rightarrow \neg b, 0.4)$ is omitted because of $Inc(K) = 0.5$.
We have:
- Plausible inferences of K are $\neg a, c \rightarrow a, b \rightarrow a, \ldots$
- Possibilistic consequences of K are $(c \rightarrow a, 0.7), (b \rightarrow a, 0.6), \ldots$.

3 Belief merging by argumentation in possibilistic logic

In this section, we consider an implementation of general framework above in order to solve the inconsistencies occur when we combine belief bases $(K_1, \ldots, K_n)$. Let us start with the concept of argument.

Definition 8. *Each argument is presented as a double $\langle S, s\rangle$, where s is a formula and S is set of formulas such that:*

(1) $S \subseteq \mathcal{K}^*$,
(2) $S \vdash s$,
(3) *S is consistent and S is minimal w.r.t. set inclusion.*

S is the support and s is the conclusion of this argument. We denote $\mathcal{A}(\mathcal{K})$ the set of all arguments built from $\mathcal{K}$.

We recall an argumentation framework in [2], it is extended from the famous one proposed by Dung in [14].

Definition 9. *An argumentation framework is a triple $\langle \mathcal{A}, \mathcal{R}, \succeq \rangle$ in which $\mathcal{A}$ is a finite set of arguments, $\mathcal{R}$ is a binary relation represented the relationship among the arguments in $\mathcal{A}$, and $\succeq$ is a preorder on $\mathcal{A} \times \mathcal{A}$. We also use $\succ$ to represent the strict order w.r.t $\succeq$.*

Definition 10. *Let X, Y be two arguments in $\mathcal{X}$.*

- *Y attacks X if $Y \succeq X$ and $Y \ \mathcal{R}\ X$.*
- *If $Y \ \mathcal{R}\ X$ but $X \succ Y$ then X can defend itself .*
- *X set of arguments $\mathcal{A}$ defends X if Y attacks X then there always exists $Z \in \mathcal{A}$ and Z attacks Y.*

Definition 11. *A set of arguments $\mathcal{A}$ is conflict-free if $\nexists X, Y \in \mathcal{A}$such that $X \ \mathcal{R}\ Y$*

The attack relations among arguments include undercut and rebut. They are defined as follows:

Definition 12. *Let $\langle S, s \rangle$ and $\langle S', s' \rangle$ be arguments of $\mathcal{A}(\mathcal{K})$. $\langle S, s \rangle$ undercuts $\langle S', s' \rangle$ if there exists $p \in S'$ such that $s \equiv \neg p$.*

Namely, an argument is under undercut attack if there exists at least one argument in its support is attacked.

Definition 13. *Let $\langle S, s \rangle$ and $\langle S', s' \rangle$ be arguments of $\mathcal{A}(\mathcal{K})$. $\langle S, s \rangle$ rebuts $\langle S', s' \rangle$ if $s \equiv \neg s'$.*

Informally, two arguments rebut each other if their conclusions are conflict.

In [1], the authors argued that each argument has a degree of influence. It allows us to compare arguments to choose the best one. When the priorities of arguments are explicit, the higher certain beliefs support, the stronger the argument is. The strength of the argument is defined as follows:

Definition 14. *The force of an argument $A = \langle S, s \rangle$, denoted by $force(A)$ is determined as follows:*

$$force(A) = min\{\alpha_i : \psi_i \in S \ and \ (\psi_i, \alpha_i) \in \mathcal{K}\}. \quad (5)$$

We consider any aggregation operator $\oplus$ satisfied the following properties:

(1) $\oplus(0, \ldots 0) = 0$,
(2) If $\alpha \geq \beta$ then for all $i = 1, \ldots, n$, then
$\oplus(x_1, \ldots, x_{i-1}, \alpha, x_{i+1}, \ldots, x_n) \geq \oplus(x_1, \ldots, x_{i-1}, \beta, x_{i+1}, \ldots, x_n)$.

Several common aggregation operators considered in literature are maximum (Max), sum (Σ) and lexicographical order($GMax$).

Proposition 1. *Let $\mathcal{K} = \{K_1, \ldots, K_n\}$ be a set of n possibilistic belief bases and $A = \langle S, s \rangle$ be an argument in $\mathcal{A}(\mathcal{K})$, then*

- $\forall \psi_i \in S, K_i \vdash (\psi_j, a_{ji}), i = 1, ..., n$.
- $force(A) = min\{\oplus(a_{j1}, \ldots, a_{jn})\}$.

By the force of argument, we can compare arguments as follows:

Definition 15. *Argument X is preferred to argument Y, denoted by $X \succ Y$ if $force(X) > force(Y)$.*

Example 4. Given $K = \{(\neg b \vee a, 0.9), (b, 0.7), (\neg d \vee a, 0.6), (d, 0.5)\}$, we have:
$K = \{(\neg b \vee a, 0.9), (b, 0.7), (\neg d \vee a, 0.6), (d, 0.5)\}$.
We have two arguments related to a :

- $A_1 =< \{\neg b \vee a, b\}, a >$,
- $A_2 =< \{\neg d \vee a, d\}, a >$.

However, A_1 is preferred to A_2 because $force(A_1) = 0.7$ and $force(A_2) = 0.5$.

The inconsistence of a possibilistic belief base K_i can be calculated from the force of inconsistent arguments as follows:

Definition 16. *Let K be a possibilistic belief base and $\langle \mathcal{A}(\mathcal{K}), Undercut, \succ\rangle$ be an argumentation framework.*

$$Inc^{att}(K) = max\{min(force(X), force(Y)) \mid \alpha_i \ att \ A_j\}. \quad (6)$$

where $att \in \{undercut, rebut\}$.

Example 5. Let $K_1 = \{(a \vee \neg b; 0.9), (f; 0.9), (g; 0.8), (\neg d \vee \neg e; 0.5), (\neg e; 0.5), (d; 0.5), (a \vee \neg d; 0.4), (\neg b \vee g; 0.3), (a \vee \neg e; 0.3), (a; 0.2), (a \vee \neg d \vee \neg e; 0.1)\}$,
$K_2 = \{(c; 0.8), (\neg f; 0.8), (\neg b \vee \neg c; 0.2), (\neg b \wedge d; 0.3)\}$, and $\oplus$ be an aggregation function defined as follows: $\oplus(\alpha, \beta) = \alpha + \beta - \alpha.\beta$. We have:
$\mathcal{K}_\oplus = \{(a \vee \neg b \vee c; 0.98), (c \vee f; 0.98), (a \vee \neg b \vee \neg f; 0.98), (c \vee g; 0.96), (\neg f \vee g; 0.96), ((a \vee \neg b) \wedge (a \vee \neg b \vee d); 0.93), ((\neg b \vee f) \wedge (d \vee f); 0.93), (a \vee \neg b \vee \neg c; 0.92), (\neg b \vee \neg c \vee f; 0.92), (a \vee \neg b; 0.9), (f; 0.9), (c \vee \neg d \vee \neg e; 0.9), (c \vee \neg e; 0.9), (c \vee d; 0.9), (\neg d \vee \neg e \vee \neg f; 0.9), (\neg e \vee \neg f; 0.9), (d \vee \neg f; 0.9), (a \vee c \vee \neg d; 0.88), (\neg b \vee c \vee g; 0.88), (a \vee \neg d \vee \neg f; 0.88), (\neg b \vee \neg f \vee g; 0.88), (a \vee c \vee \neg e; 0.86), ((g \vee \neg b) \wedge (g \vee d); 0.86), (a \vee \neg e \vee \neg f; 0.86), (a \vee c; 0.84), (\neg b \vee \neg c \vee g; 0.84), (a \vee \neg f; 0.84), (a \vee c \vee \neg d \vee \neg e; 0.82), (a \vee \neg d \vee \neg e \vee \neg f; 0.82), (g; 0.8), (c; 0.8), (\neg f; 0.8), ((\neg b \vee \neg e) \wedge (d \vee \neg e); 0.65), ((\neg b \vee d) \wedge (d); 0.65), (\neg b \vee \neg c \vee \neg d \vee \neg e; 0.6), (\neg b \vee \neg c \vee \neg e; 0.6), (\neg b \vee \neg c \vee d; 0.6), (a \vee \neg b \vee \neg c \vee \neg d; 0.52), ((\neg b \vee g) \wedge (\neg b \vee g \vee d); 0.51), ((a \vee \neg b \vee \neg e) \wedge (a \vee d \vee \neg e); 0.51), (\neg d \vee \neg e; 0.5), (\neg e; 0.5), (d; 0.5), (\neg b \vee \neg c \vee g; 0.44), (a \vee \neg b \vee \neg c \vee \neg e; 0.44), ((a \vee \neg b) \wedge (a \vee d); 0.44), (a \vee \neg d; 0.4), (\neg b \vee g; 0.3), (a \vee \neg e; 0.3), (\neg b \wedge d; 0.3), (a \vee \neg b \vee \neg c \vee \neg d \vee \neg e; 0.28), (a; 0.2), (\neg b \vee \neg c; 0.2), (a \vee \neg d \vee \neg e; 0.1)\}$.
Table 1 is the set of arguments built from $\mathcal{K}_\oplus$ an their force. We have:
$Undercut = (A_{11}, A_{32}), (A_{11}, A_{33}), (A_{32}, A_{11}), (A_{32}, A_{12}), (A_{32}, A_{16}), (A_{32}, A_{17}), (A_{32}, A_{18}), (A_{32}, A_{19}), (A_{32}, A_{21}), (A_{32}, A_{22}), (A_{32}, A_{25}), (A_{32}, A_{28}), (A_{32}, A_{29}), (A_{32}, A_{30})$.

We have:
$Inc^{undercut}(\mathcal{K}_\oplus) = max\{min(0.9, 0.8), min(0.9, 0.8), min(0.8, 0.9), min(0.8, 0.9), min(0.8, 0.9), min(0.8, 0.9), min(0.8, 0.9), min(0.8, 0.88), min(0.8, 0.88), min(0.8, 0.88), min(0.8, 0.86), min(0.8, 0.84), min(0.8, 0.82), min(0.8, 0.82)\} = 0.8$.
Therefore, the inconsistency degree of $\mathcal{K}_\oplus$ is 0.8.

Now, we can define the belief merging by argumentation as follows:

Definition 17. *Let $\mathcal{K} = \{K_1, \ldots, K_n\}$ be a set of possibilistic belief bases. Belief merging operator is defined as follows:*
$\Delta^{att}_\oplus(\mathcal{K}) = \{\psi | (\psi, a) \in \mathcal{K}_\oplus, a > Inc^{att}(\mathcal{K}_\oplus)\}$ *where $att \in \{indercut, rebut\}$.*

Argument	force
$A_1 =< \{a \vee \neg b \vee c\}, a \vee \neg b \vee c >$	0.98
$A_2 =< \{c \vee f\}, c \vee f >$	0.98
$A_3 =< \{a \vee \neg b \vee \neg f\}, a \vee \neg b \vee \neg f >$	0.98
$A_4 =< \{c \vee g\}, c \vee g >$	0.96
$A_5 =< \{\neg f \vee g\}, \neg f \vee g >$	0.96
$A_6 =< \{(a \vee \neg b) \wedge (a \vee \neg b \vee d)\}, (a \vee \neg b) \wedge (a \vee \neg b \vee d) >$	0.93
$A_7 =< \{((\neg b \vee f) \wedge (d \vee f))\}, (\neg b \vee f) \wedge (d \vee f) >$	0.93
$A_8 =< \{a \vee \neg b \vee \neg c\}, a \vee \neg b \vee \neg c >$	0.92
$A_9 =< \{\neg b \vee \neg c \vee f\}, \neg b \vee \neg c \vee f >$	0.92
$A_{10} =< \{a \vee \neg b\}, a \vee \neg b >$	0.9
$A_{11} =< \{f\}, f >$	0.9
$A_{12} =< \{f, \neg f \vee g\}, g >$	0.9
$A_{13} =< \{c \vee \neg d \vee \neg e\}, c \vee \neg d \vee \neg e >$	0.9
$A_{14} =< \{c \vee \neg e\}, c \vee \neg e >$	0.9
$A_{15} =< \{c \vee d\}, c \vee d >$	0.9
$A_{16} =< \{\neg d \vee \neg e \vee \neg f, f\}, \neg d \vee \neg e >$	0.9
$A_{17} =< \{\neg e \vee \neg f, f\}, \neg e >$	0.9
$A_{18} =< \{d \vee \neg f, f\}, d >$	0.9
$A_{19} =< \{a \vee c \vee \neg d, d \vee \neg f, f\}, a \vee c >$	0.88
$A_{20} =< \{\neg b \vee c \vee g\}, \neg b \vee c \vee g >$	0.88
$A_{21} =< \{a \vee \neg d \vee \neg f, f\}, a \vee \neg d >$	0.88
$A_{22} =< \{\neg b \vee \neg f \vee g, f\}, \neg b \vee g >$	0.88
$A_{23} =< \{a \vee c \vee \neg e\}, a \vee c \vee \neg e >$	0.86
$A_{24} =< \{(g \vee \neg b) \wedge (g \vee d)\}, (g \vee \neg b) \wedge (g \vee d) >$	0.86
$A_{25} =< \{a \vee \neg e \vee \neg f, f\}, a \vee \neg e >$	0.86
$A_{26} =< \{a \vee c\}, a \vee c >$	0.84
$A_{27} =< \{\neg b \vee \neg c \vee g\}, \neg b \vee \neg c \vee g >$	0.84
$A_{28} =< \{a \vee \neg f, f\}, a >$	0.84
$A_{29} =< \{a \vee \neg d \vee \neg e \vee \neg f, f\}, a \vee \neg d \vee \neg e >$	0.82
$A_{30} =< \{a \vee \neg d \vee \neg e \vee \neg f, f, d \vee \neg f\}, a \vee \neg d \vee \neg f >$	0.82
$A_{31} =< \{c\}, c >$	0.8
$A_{32} =< \{\neg f\}, \neg f >$	0.8
$A_{33} =< \{\neg b \vee \neg c \vee f, c, \neg f\}, \neg b >$	0.8
$A_{34} =< \{\neg b \vee \neg c \vee g, c\}, \neg b \vee g >$	0.84
$A_{35} =< \{(\neg b \vee \neg e) \wedge (d \vee \neg e)\}, (\neg b \vee \neg e) \wedge d \vee \neg e) >$	0.65
$A_{36} =< \{(\neg b \vee d) \wedge (d)\}, (\neg b \vee d) \wedge (d) >$	0.65
$A_{37} =< \{\neg b \vee \neg c \vee \neg e, c\}, \neg b \vee \neg e >$	0.6
$A_{38} =< \{\neg b \vee \neg c \vee d, c\}, \neg b \vee d >$	0.6
$A_{39} =< \{(\neg b \vee g) \wedge (\neg b \vee g \vee d)\}, (\neg b \vee g) \wedge (\neg b \vee g \vee d) >$	0.51
$A_{40} =< \{(a \vee \neg b \vee \neg e) \wedge (a \vee d \vee \neg e)\}, (a \vee \neg b \vee \neg e) \wedge (a \vee d \vee \neg e) >$	0.51
$A_{41} =< \{a \vee \neg b \vee \neg c \vee \neg e, c\}, a \vee \neg b \vee \neg e >$	0.44
$A_{42} =< \{(a \vee \neg b) \wedge (a \vee d)\}, (a \vee \neg b) \wedge (a \vee d) >$	0.44

Table 1. Forces of arguments.

We call $\Delta_{\oplus}^{att}$ the family of BMA (*Belief Merging by Argumentation*) operators.

Example 6. Continuing Example 5, with $att = undercut$ and $\oplus(\alpha, \beta) = \alpha + \beta - \alpha.\beta$ we have:
$\Delta_{\oplus}^{att}(\mathcal{K}) = \{\{(a \vee \neg b \vee c), (c \vee f), (a \vee \neg b \vee \neg f), (c \vee g), (\neg f \vee g), ((a \vee \neg b) \wedge (a \vee \neg b \vee d)), ((\neg b \vee f) \wedge (d \vee f)), (a \vee \neg b \vee \neg c), (\neg b \vee \neg c \vee f), (a \vee \neg b), (f), (c \vee \neg d \vee \neg e), (c \vee \neg e), (c \vee d), (\neg d \vee \neg e \vee \neg f), (\neg e \vee \neg f), (d \vee \neg f), (a \vee c \vee \neg d), (\neg b \vee c \vee g), (a \vee \neg d \vee \neg f), (\neg b \vee \neg f \vee g), (a \vee c \vee \neg e), ((g \vee \neg b) \wedge (g \vee d)), (a \vee \neg e \vee \neg f), (a \vee c), (\neg b \vee \neg c \vee g), (a \vee \neg f), (a \vee c \vee \neg d \vee \neg e), (a \vee \neg d \vee \neg e \vee \neg f)\}$.

4 Postulates and logical properties

We recall that $\mathcal{K} = \{K_1, \ldots, K_n\}$ is a finite set of possibilistic belief bases, AF_s is an argumentation framework is determined from $\mathcal{K}$. Aggregation function $\mathcal{K}_{\oplus}$ is defined as follows: $\mathcal{K}_{\oplus} : \mathbb{K}^n \to \mathbb{K}^*$. The set of postulates is introduced as follows:

(SYM) $\mathcal{K}_{\oplus}(\{K_1, \ldots, K_n\}) = \mathcal{K}_{\oplus}(\{K_{\pi(1)}, \ldots, K_{\pi(n)}\})$, where π is a permutation in $\{1, \ldots, n\}$.
Postulate (SYM), sometimes called (ANON)[12], ensures the equity of participants. It states that the result of an argumentation process should reflect the arguments of the participants rather than their identity.

(CON) $\nexists \psi \in \mathcal{L}(\mathcal{K}_{\oplus}(\{K_1, \ldots, K_n\}) \vdash \psi) \wedge (\mathcal{K}_{\oplus}(\{K_1, \ldots, K_n\}) \vdash \neg\psi)$
Postulate (CON) states that belief merging by argumentation should return a consistent result.

(UNA) if $K_1^* \equiv \ldots \equiv K_n^*$ then $\mathcal{K}_{\oplus}(\{K_1, \ldots, K_n\}) \equiv K_1^*$.
Postulate (UNA) presents the assumption of unanimity. It states that if all participants possess the same set of beliefs, then this set of belief should be the result of argumentation process. Clearly, Postulate (UNA) is more general than postulate (IDN) and it also implies (IDN) which is defined as follows:

(IDN) $\mathcal{K}_{\oplus}(\{K_i, \ldots, K_i\}) \equiv K_i^*$
It states that if all participants have the same possibilistic belief base, then after the argumentation process, we should have the result as its associated belief base.

(CLO) $\cup_{i=1}^n B_i^* \vdash \mathcal{K}_{\oplus}(\{K_i, \ldots, K_i\})$
Postulate (CLO) requires the closure of the result of argumentation process. It states that any belief in argumentation result should be in at least some input belief base.

(MAJ) if $|\{K_i^* \vdash \psi, i = 1 \ldots n\}| > \frac{n}{2}$ then $\mathcal{K}_{\oplus}(\{K_i, \ldots, K_i\}) \vdash \psi$.
Postulate (MAJ) states that if a belief is supported by the majority group of participants, it should be in the result of argumentation process.

(COO) if $K_i^* \vdash \psi, i = 1 \ldots n$ then $\mathcal{K}_{\oplus}(\{K_i, \ldots, K_i\}) \vdash \psi$.
Postulate (COO) states that if a belief is supported by all participants, it should be in the result of argumentation process.

We have the following lemma:

Lemma 2. *It holds that:*

- *(UNA) implies (IDN);*
- *(MAJ) implies (COO).*

Investigate the properties of belief merging operator defined in the previous section we have:

Theorem 1. *Family of BMA operators satisfies the following postulates* (SYM), (CON)*,* (UNA)*, and* (CLO)*. It does not satisfy* (MAJ)*.*

5 Conclusion

In this paper, a framework for merging possibilistic belief bases by argumentation is introduced and discussed. The key idea in this work is using the inconsistent degree as a measure together with the notion of undercut to construct an argumentation framework for belief merging. A set of postulates is introduced and logical properties are mentioned and discussed. They assure that the proposed model is sound and complete. The deeper analysis on the set of postulates and logical properties, and the evaluation of computational complexities of belief merging operators in this framework are reserved as future work.

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