Hesitant Bipolar Fuzzy MCDM Framework for Evaluating Swimming Analysis Technologies

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The analysis of swimming techniques has become increasingly significant for enhancing performance metrics and optimizing training methods. This study presents a novel approach to evaluate and select the optimal technology for swimming technique analysis by employing a Multi-Criteria Decision-Making (MCDM) framework within a hesitant bipolar fuzzy environment. Traditional evaluation methods often fail to handle expert evaluations' inherent uncertainty and hesitation. To address this gap, our approach integrates hesitant bipolar fuzzy sets, effectively capturing expert judgements with high precision and flexibility. Through this method, we assess a range of technological tools across multiple criteria, including accuracy, usability, affordability, and real-time feedback capabilities. The results reveal that the chosen MCDM model achieves an accuracy of 99.2% in aligning with expert preferences, establishing it as a reliable method for ranking swimming analysis technologies. Moreover, our findings indicate that Technology D outperforms others with a preference score of 0.90, suggesting its suitability for extensive application in sports training environments. This study not only highlights the effectiveness of hesitant bipolar fuzzy sets in sports technology evaluation but also provides a robust framework for similar applications across other domains where decision-making under uncertainty is critical.

Prispevek predstavi nov okvir za oceno tehnologij analize plavalne tehnike, ki uporablja MCDM metodo v okolju z negotovostjo in bipolarno zamegljenostjo. Ta pristop učinkovito obravnava negotovost in dvome strokovnjakov ter natančno oceni različne tehnologije na podlagi več kriterijev.

1 Introduction

Swimming has become a significant area in sports science, and applying technology in performance analysis could greatly benefit the elite sports performer and the coach [1]. The application of technology in handling swimming skills and styles is of particular importance and relevance, where better strategies can be developed, or wrong ones are removed, and the biomechanics and postural efficiency of movements are enhanced [2]. Hence, choosing this particular technology for the above-mentioned purpose is considered essential but not easy because of the variety of technologies, and selecting the best among them implies the problems of defining the performance evaluation criteria [3] [4].

In recent years, multi-criteria decision-making (MCDM) models have gained recognition as promising tools in assessing technologies since they enable decision-making based on several criteria [5]. However, most previous works in the MCDM area fail to capture the inherent stochasticity and conservatism that usually accompany the rating process, mainly when the domain highly depends on an expert's opinion [6]. Regarding this, hesitant bipolar fuzzy sets (HBFS), a new acquisition to the fuzzy set theory, have proved to apply these subjective factors more efficiently since the HBFS capture both positive and negative aspects of the experts [7]. This study uses HBFS for the first time in the MCDM process to overcome the challenges caused by evaluating swimming analysis technology, making it unique [8].

1.1 Research gap

Recent works in sports science and technology literature on performance analysis examine different methods where tools include wearable technologies, video technology systems, and biomechanical models [9]. However, the models employed to assess and validate these tools' readiness potential and make decisions regarding selecting appropriate technology depend on the conventional MCDM techniques, including the analytic hierarchy process (AHP), the technique for order preference by similarity to the ideal solution (TOPSIS), and many other similar models [10]. These methods have limitations when applied to expert evaluations in complicated sports environments [11]. Firstly, most traditional MCDM approaches have drawbacks in solving the issues of hesitation and bipolarity of specialists' opinions. The specialists might be cautious when delivering quantitative assessments, even when new technologies or unknown approaches are used [12]. Furthermore, in the evaluation of technology related to sports analysis, it can be observed that the perceptions of specialists consist of value judgments that contain positive and negative elements that differ with respect to the criteria, which facts substantiate the bipolar character of the judgment and are not adequately incorporated into conventional methods [13]. The above-cited studies, therefore, failed to provide comprehensive coverage of the whole range of subjectively perceived factors essential when selecting the most suitable technologies for swimming technique analysis, which points to a research gap [14]. A key objective is to demonstrate the model's capacity to replicate expert preferences with high accuracy. As detailed in the results, the proposed method achieves a 99.2% alignment with expert rankings, indicating its robustness and applicability for real-world decision support in sports technology assessments.

1.2 Limitations of previous studies

Previous research on technology evaluation in sports science has encountered several notable limitations:

1. Inability to capture expert hesitation: Standard MCDM frameworks assume that experts provide definitive judgements, overlooking the reality that experts may feel hesitant in ranking or scoring certain technologies due to limited familiarity or mixed feelings about specific tools.

2. Lack of flexibility in decision modeling: The absence of advanced fuzzy logic in conventional models restricts their capacity to adapt to varied, subjective evaluations that experts may provide, particularly in settings involving innovative or lesser-known technologies.

3. Insufficient support for bipolar opinions: Traditional MCDM approaches, which rely on single-directional preference scales, lack the functionality to handle bipolarity, where experts simultaneously consider the positive and negative aspects of each option. This limitation can lead to overly simplistic evaluations that fail to reflect the true complexity of expert opinions.

4. Low accuracy in reflecting expert preferences: As a consequence of the above limitations, previous frameworks have demonstrated lower alignment with actual expert preferences, reducing the reliability of the decision-making process.

Given these limitations, the current study proposes a hesitant bipolar fuzzy MCDM framework to enhance technology evaluation's flexibility, precision, and accuracy in the context of swimming technique analysis. The main data source for evaluating swimming analysis technologies consists of expert assessments, which rate accuracy and usability, and feedback quality and cost-effectiveness. The experts assign their ratings regarding the domain based on their knowledge, and then the model uses hesitant bipolar fuzzy numbers to capture their dual sentiments and unclearness. The initial fuzzy evaluations provided by experts serve as the fundamental information source for an MCDM process using HBFS to produce final alternative rankings. Expert opinions enter directly into the model without altering their initial hesitancy through this structure.

1.3 Challenges of the study

Conducting a comprehensive evaluation of swimming analysis technologies through HBFS MCDM presents distinct challenges that are crucial to address for effective model implementation and reliable results. These challenges include:

– Data collection challenges: Collecting detailed, reliable feedback from domain experts, particularly in fields as specialized as swimming performance analysis, requires careful consideration of expert background, expertise level, and subjective bias. Experts may have varying familiarity levels with different technologies, further complicating feedback consistency.

– Handling uncertainty in expert judgments: A core challenge in using HBFS MCDM is managing uncertainty effectively. Experts may not provide entirely definitive judgements due to uncertainty in the evaluation criteria or unfamiliarity with some technologies. HBFS offers a mechanism for handling such uncertainty but requires careful parameterization to ensure accurate representation.

– Computational complexity and model feasibility: Although HBFS MCDM models enhance the decision-making process, they also introduce computational complexities that make them difficult to apply in practice. For this model to be feasible in real-world scenarios, careful calibration is needed to balance computational efficiency and decision accuracy.

Addressing these challenges is essential for implementing an effective HBFS MCDM model, ensuring it achieves the desired accuracy and reliability in technology evaluation.

1.4 Motivation

The primary purpose of this paper is to help fill the research gap and provide a more elaborate and accurate decision-supported view on the evaluation of sports technology. Exploring HBFS in this study propels sports analysis by improving the credibility of decision, while reflecting the uncertainty and subjectivity of expert judgements in decision making about technology adoption. This study aims to obtain an optimal solution for evaluating tools for analyzing swimming technique styles by utilizing advanced fuzzy set theory in a multicriteria decision-making system. The flexible structure of the framework benefits the broadening use of the framework in various sports and for technology assessment, and therefore, it spurs its development and research.

1.5 Novel contributions

This research introduces several novel contributions to the field of sports science and technology evaluation:

1. Application of hesitant bipolar fuzzy sets in sports analysis technology evaluation: This study pioneers HBFS within an MCDM framework, offering a unique approach for accurately capturing the complexities of expert opinions in sports technology evaluation.

2. Development of a specialized MCDM model for swimming technique analysis: By integrating HBFS into an MCDM model tailored for swimming technique analysis, this research addresses the unique requirements of the sport, including multi-dimensional performance criteria, uncertainty in expert judgement, and the need for high-accuracy decision support.

3. Empirical validation of decision accuracy: Through rigorous testing and validation, the model demonstrates a decision accuracy of 99.2%, substantiating its efficacy in reflecting expert preferences and improving existing evaluation methods.

These contributions underscore the originality of this study and its relevance to the broader field of sports technology evaluation, where decision-making under uncertainty is paramount.

The remainder of this paper is organized as follows: Section 2 reviews existing literature on MCDM methods, hesitant fuzzy sets, and sports technology evaluation, highlighting relevant studies and theoretical underpinnings. Section 3 details the Methodology used in the study, describing the integration of HBFS within the MCDM framework and the criteria considered for swimming analysis technology evaluation. Section 4 presents the Experimental Results and Analysis, showcasing model outcomes, accuracy rates, and comparative assessments against other decision-making frameworks. Section 5 provides a discussion on the Implications and Future Research Directions, suggesting areas for further exploration and practical applications of the proposed model. Finally, Section 6 concludes the study, summarizing key findings and reaffirming the contributions made to the field.

2 Literature review

The evaluation of advanced decision-making frameworks in diverse domains continues to gain significance, as it offers insights into addressing complex challenges with precision. This section explores key studies that highlight innovative approaches to multi-criteria decision-making (MCDM) and their applications in various fields.

Ali et al. [15] introduced a new method for solving multi-faceted decision-making issues, which can be especially useful in economic matters, energy supply and demand challenges, and the population's resource scarcity. To develop more effective models for solving complex problems, their study proposed the Spherical Fuzzy Bipolar Soft Sets (SFBSSs) model. It was suggested that this model be used instead of the proposed spherical fuzzy set hybridizations because those do not handle information equally in a bipolar setting. They provided empirical evidence of SFBSSs and showed how such models could be used by working through a real-life corporate decision-making problem—the selection of a chief management officer. Their research also looked at other features and functions of SFBSSs, such as subset, complement, relative null and absolute set, extended union and intersection, and restricted union and intersection [16]. To explain why operations like AND and OR are valid, primary number results like commutativity, associativity, and distribution, along with De Morgan's laws, were used in the context of the SFBSS environment. Additionally, they studied a multiple-attribute decision approaching hierarchy ranking downstream fish passage designs for hydroelectric utilities where the objectives reflected an optimal tradeoff between the hydropower and ecological impacts on fish migration. Their comparison established the usefulness of the SFBSS model in outcompeting other approaches; it is also invariant to negative, neutral, and positive memberships under volatile conditions.

In a multicriteria assessment of technologies of seawater electrolysis for green hydrogen production at sea, D'Amore-Domenech et al. [17] focused on the benefits of using maritime renewable sources for power production. Nevertheless, several benefits, marine renewables, when combined with electrolysis technology, remain unprofitable for commercial purposes. The study's goal was to find out which of the listed electrolysis technologies looked most promising based on economic, environmental, and social approaches, given that it is often difficult to achieve the best result in all the aspects listed above. To accommodate this, the researchers used multicriteria decision-making (MCDM) techniques, and while its application is efficient, it sometimes serves as a source of incoherent analysis. To overcome this, the study used five different MCDM techniques, and the reliability of the results was boosted by ensuring that the ranking algorithms were consistent. A survey analysis of the study pointed out that PEM electrolysis suits seawater electrolysis in the short run, as demonstrated by its provision of a reasonable opportunity for green hydrogen application in combination with marine renewable sources.

Abdullah et al. [18] proposed the establishment of a causal relationship between criteria influencing water security based on the intuitive fuzzy decision-making trial and evaluation laboratory (IF-DEMATEL) technique. This work differs from the basic concept of DEMATEL by using

IFNs instead of crisp numbers because the degree of hesitation is inherent in the experts' estimations. According to the mentioned variables, influences were collected from the water security professionals through one-to-one interviews concerning seven criteria in water security using the three tuples of IFNs. Operating IF-DEMATEL through specialized software enabled the computational aspect, producing a causal relationship map. The evaluation concluded that "over-abstraction," "saltwater intrusion," and "limited infrastructures" were initial causes of water insecurity and that "water pollution" and "rapid urbanization" were primary criteria most sensitive to other circumstances in the system. Thus, the study's findings can help practice water security management and generate research on using modified DEMATEL with IFNs, illustrating critical issues for policymakers.

Du and Yang [19] introduced the method of advanced market risk assessment of SMEs based on the IVIFHIPG technique. This method cannot only solve the problem that SMEs' development scale and system are often limited in China but also can not form competitive strength and sustainable development capability. Recognising the centrality of risk management, the authors defined market risk evaluation as a multiple attribute decision making (MADM) problem under uncertainty. To capture uncertainty, the authors used interval-valued intuitive fuzzy sets (IVIFSs), which provide a means for expressing uncertain data in the context of market risk evaluation [20]. Explorations of the options and features of the IVIFHIPG technique were made, and a case study was presented to demonstrate the technique's effectiveness in SMEs' market risk appraisal. The main contributions of the study are the development of the IVIFHIPG model, demonstration of its practical usage for evaluating market risk, carrying out comparative analysis to determine the efficiency of the method and thus the applicability of various risk assessments under uncertainty for SMEs, and proposing the IVIFHIPG to support SMEs in intensively competitive markets.

Mao [21] came up with a more sophisticated method to gauge the operational effectiveness of businesses that combine industry and finance using the Interval-Valued Intuitionistic Fuzzy Hamacher Interactive Power Averaging (IVIFHIPA) technique. Given the rising competitive pressures experienced by enterprises as a result of economic globalization, enterprises' financial management faces pressures toward change [22]. This study integrates industry finance, an emerging strategy that seeks to improve the effectiveness of financial management and control, reduce risks, and improve the capacity of industries. He deliberated the operational quality evaluation of such enterprises as a multiple attribute decision-making (MADM) problem under uncertainty with the help of IVIFSs to handle vague and uncertain information. So, the IVIFHIPA technique was created to combine the Interval-Valued Intuitionistic Fuzzy Hamacher Interactive Weighted Averaging method with the traditional power average method. It is more accurate and flexible than MADM processes. The IVIFHIPA technique was evaluated in terms of its properties and parameters, and it was tested with a real-life example of evaluating operational quality for combining finance and industry using lean management accounting. He pioneered the IVIFHIPA model's development, validation, and use to increase operational quality assessments in complex, interfaced financial systems.

The literature reveals significant advancements in decision-making frameworks, addressing various challenges across diverse applications. By analyzing these studies, this paper positions itself to build on existing methodologies while addressing unresolved gaps, thereby advancing the domain of multi-criteria decision-making. Table 1 provides the comparison of state-of-the-art methods for swimming technology evaluation.

3 Methodology

This research establishes a method for analyzing and comparing the best technology for swimming technique analysis based on a multi-criteria decision-making (MCDM) application under the hesitant bipolar fuzzy context. This is so because the methodology adopts hesitant bipolar fuzzy sets (HBFS) together with MCDM to deal with the uncertain nature of the expert assessments where both the positive and negative parts of the subjective assessments are captured. By combining fuzzy set theory and MCDM, the approach emphasizes the technologies based on criteria like accuracy, usability, economic feasibility, and feedback quality, which are required to judge the swimming analysis tools. It presents a transparent and integrated framework that can address decision-making problems in situations that require defuzzified but subtly different expert opinions.

3.1 Mathematical foundation of the model

3.1.1 Hesitant bipolar fuzzy sets (HBFS)

Hesitant bipolar fuzzy sets (HBFS) offer a mathematical structure to handle complex evaluations involving both hesitation and bipolarity, representing positive and negative opinions about a given attribute. For an attribute x in an HBFS A, the membership $\mu_A(x)$ and non-membership $\nu_A(x)$ degrees are defined as intervals:

$$\mu_A(x) = [\mu_A^L(x), \mu_A^U(x)]$$
$$\nu_A(x) = [\nu_A^L(x), \nu_A^U(x)]$$

where $\mu_A^L(x)$ and $\mu_A^U(x)$ are the lower and upper bounds of the membership interval, while $\nu_A^L(x)$ and $\nu_A^U(x)$ represent the bounds of the non-membership interval. The hesitation degree $\pi_A(x)$ reflects the uncertainty and is calculated as:

$$\pi_A(x) = 1 - \mu_A^U(x) - \nu_A^U(x) \quad (1)$$

Table 1: Comparison of state-of-the-art methods for swimming technology evaluation

Method	Performance Metrics	Limitations	Suitability for Swimming Analysis
AHP [13]	Accuracy, Usability, Cost-effectiveness	Fails to capture hesitation and bipolarity in expert opinions	Suitable for general MCDM but inadequate for capturing expert uncertainty in sports contexts
TOPSIS [?]	Performance alignment with ideal solution, Usability	Assumes crisp judgements, lacks flexibility for complex decisions	Limited in dealing with subjective or ambiguous expert feedback
Fuzzy AHP [?]	Accuracy, Decision support efficiency	Does not adequately address uncertainty in expert judgement	Can be used but does not fully integrate the complexities of hesitant and bipolar evaluations
Spherical Fuzzy Sets [15]	Robust decision-making in uncertain environments	Inability to reflect both positive and negative aspects of expert opinions	Limited in addressing both the positive and negative dimensions required in technology evaluation
Our Method (HBFS MCDM)	99.2% accuracy, Flexibility in expert evaluation, Real-time applicability	Computational complexity, Need for expert calibration	Fully captures expert hesitation and bipolarity, addresses gaps in previous methods by offering a flexible, high-accuracy framework

This hesitation component provides a nuanced approach to handling ambiguous expert judgments, where opinions may not be entirely positive or negative.

3.1.2 Bipolar fuzzy aggregation

In the evaluation process, hesitant bipolar fuzzy aggregation captures expert preferences by adjusting the interaction between membership and non-membership values. For example, combining two HBFS A and B with membership and non-membership intervals can be achieved using specific aggregation operators:

$$\mu_{A\cap B}(x) = \frac{\mu_A(x)\cdot\mu_B(x)}{\lambda + (1-\lambda)(\mu_A(x)+\mu_B(x)-\mu_A(x)\cdot\mu_B(x))} \tag{2}$$

$$\nu_{A\cup B}(x) = \frac{\nu_A(x)+\nu_B(x)-\nu_A(x)\cdot\nu_B(x)}{\lambda + (1-\lambda)\cdot(\nu_A(x)+\nu_B(x)-\nu_A(x)\cdot\nu_B(x))} \tag{3}$$

where λ is the interaction parameter that controls the level of influence between the attributes.

3.1.3 Illustrative example of HBFS aggregation

To enhance understanding of the hesitant bipolar fuzzy weighted averaging (HBFWA) operator, we present a simple numerical example. Suppose we have three hesitant bipolar fuzzy elements (HBFEs) associated with a criterion:

– $h_1 = \{(0.6, -0.2), (0.5, -0.1)\}$

– $h_2 = \{(0.7, -0.3)\}$

– $h_3 = \{(0.4, -0.4), (0.5, -0.2)\}$

with corresponding weights:

$$w_1 = 0.3, \quad w_2 = 0.4, \quad w_3 = 0.3$$

First, compute the average positive and negative membership values for each HBFE:

$$\text{avg}^+_{h_1} = \frac{0.6+0.5}{2} = 0.55, \quad \text{avg}^-_{h_1} = \frac{-0.2+(-0.1)}{2} = -0.15$$

$$\text{avg}^+_{h_2} = 0.7, \quad \text{avg}^-_{h_2} = -0.3$$

$$\text{avg}^+_{h_3} = \frac{0.4+0.5}{2} = 0.45, \quad \text{avg}^-_{h_3} = \frac{-0.4+(-0.2)}{2} = -0.3$$

Now, aggregate the values using the weighted average:

$$\begin{aligned}
\mu^+ &= w_1\cdot 0.55 + w_2\cdot 0.7 + w_3\cdot 0.45 \\
&= 0.165 + 0.28 + 0.135 = 0.58 \\
\mu^- &= w_1\cdot(-0.15) + w_2\cdot(-0.3) + w_3\cdot(-0.3) \\
&= -0.045 - 0.12 - 0.09 = -0.255
\end{aligned}$$

Thus, the aggregated HBFE is:

$$h^* = (0.58, -0.255)$$

This step-by-step example clarifies how hesitant bipolar fuzzy information is combined using the HBFWA operator, as employed in the proposed decision-making framework.

3.2 Construction of the MCDM model for swimming technology evaluation

The model employs the hesitant bipolar fuzzy interactive averaging (HB-FIA) technique to evaluate criteria for swimming technology. Each technology is evaluated by weighting attributes such as accuracy, usability, cost-effectiveness, and feedback quality. Given a set of attributes $A_1, A_2, \ldots, A_n$ and weights $w_1, w_2, \ldots, w_n$, the aggregated values are calculated as follows:

$$HB\text{-}FIA(A_1, A_2, \ldots, A_n) = \left(\prod_{i=1}^{n} \mu_{A_i}(x)^{w_i}\right)^{\frac{1}{\sum_{i=1}^{n} w_i}}, \left(\prod_{i=1}^{n} \nu_{A_i}(x)^{w_i}\right)^{\frac{1}{\sum_{i=1}^{n} w_i}} \quad (4)$$

The HB-FIA technique allows for weighted aggregation, balancing each attribute's impact according to its importance in the decision process. A total of four evaluation factors were chosen to assess swimming analysis technologies: accuracy combined with usability and cost-effectiveness along with feedback quality. The accuracy rate is essential to conduct proper performance assessments and correct techniques. Both athletes and coaches can easily use the technology due to its usability design characteristics that eliminate the need for extensive training or technical support. The price of the tools stands as a crucial factor for allowing institutions and individuals who have limited resources to access them. The quality of feedback demanded by athletes and coaches needs to be high in order to provide performance-enhancing insights on schedule. Cognitive metrics that emphasize resistance against diverse environmental aspects, including water turbulence, pool configurations and lighting variations, were added as supplemental evaluation criteria. This upgrade considers realistic operational obstacles affecting these systems in the field, enabling enhanced evaluation framework comprehensiveness.

3.2.1 Properties of the HB-FIA technique

- Sensitivity to Attribute Interactions: The HB-FIA technique accounts for weighted influences, making it adaptable to various levels of attribute significance.
- Enhanced Decision Precision: By integrating HBFS, the model effectively represents both positive and negative judgments across criteria, capturing the full spectrum of expert opinions.

3.2.2 Parameterization of HBFS model

To ensure replicability and procedural transparency, we define the parameterization steps adopted in applying the HBFS-based MCDM framework. First, expert weights were obtained using a linguistic scale mapped to triangular fuzzy numbers, which were subsequently converted into normalized crisp values via a defuzzification process. Second, for each criterion, experts provided a set of bipolar hesitant values representing both positive and negative membership degrees. These values were aggregated using the HBFS averaging operator. The hesitation degrees were constructed by recording multiple values for each expert's judgment under uncertainty. Each set was transformed into a bipolar structure, where the positive set indicated support and the negative set indicated opposition to an alternative under a specific criterion. A threshold τ was set at 0.5 to distinguish between dominant and non-dominant evaluations, and normalization was applied across all criteria to maintain comparability.

3.3 Algorithm for implementing the HB-FIA model

The following algorithm details the steps involved in using the HB-FIA model for ranking swimming analysis technologies:

Algorithm 1 Detailed Implementation of the HB-FIA Method

Require: Expert evaluations $E = \{e_1, e_2, \ldots, e_n\}$ under criteria $C = \{c_1, c_2, \ldots, c_m\}$
Ensure: Final ranking of alternatives
1: **Step 1:** Normalize the hesitant bipolar fuzzy evaluations under each criterion.
2: **Step 2:** Construct hesitant bipolar fuzzy decision matrix $D = [d_{ij}]$, where d_{ij} represents the positive and negative membership degrees for alternative i on criterion j.
3: **Step 3:** Compute criterion weights w_j either via expert-assigned values or using entropy/objective methods. For this study, expert-assigned weights reflecting real-world preference sensitivity are used.
4: **Step 4:** Apply aggregation operator (e.g., HBFAWA) on D using weights w_j to obtain aggregate scores for each alternative.
5: **Step 5:** Defuzzify the aggregate hesitant bipolar fuzzy values to obtain crisp utility values.
6: **Step 6:** Rank alternatives based on defuzzified values.

3.3.1 Pseudo-code of the HBFS-MCDM algorithm

Note on Weighting Strategy: The weights assigned to each criterion reflect expert judgments on their relative importance. Since these are inherently subjective, the model integrates them proportionally into the aggregation process to preserve the integrity of domain-specific knowledge. This approach aligns with the principle of preference-sensitive decision-making often required in expert-driven sports technology evaluations.

Algorithm 2 HBFS-MCDM Algorithm

1: **Input:** Set of alternatives $A = \{A_1, A_2, \ldots, A_m\}$, criteria $C = \{C_1, C_2, \ldots, C_n\}$, weights w_j, and hesitant bipolar fuzzy decision matrix $D = [h_{ij}]$
2: **for** each alternative A_i **do**
3: **for** each criterion C_j **do**
4: Extract HBFE h_{ij} and compute average positive and negative membership values: μ_{ij}^+, μ_{ij}^-
5: **end for**
6: **end for**
7: **Aggregation:** Apply HBFWA or HBFPWA to obtain $h_i^* = (\mu_i^+, \mu_i^-)$ for each A_i
8: **Scoring:** Compute score function $S(h_i^*) = \mu_i^+ + \mu_i^-$
9: **Ranking:** Rank all alternatives based on descending order of $S(h_i^*)$
10: **Output:** Ranked list of alternatives

3.4 Illustrative example of HB-FIA application

To demonstrate the model, we consider three key attributes in the context of swimming technology: accuracy, usability, and feedback quality. Suppose the membership and non-membership intervals for each attribute are as follows:

$$\mu_{A_1}(x) = [0.7, 0.9], \quad \nu_{A_1}(x) = [0.1, 0.2]$$
$$\mu_{A_2}(x) = [0.6, 0.8], \quad \nu_{A_2}(x) = [0.2, 0.3]$$
$$\mu_{A_3}(x) = [0.8, 0.95], \quad \nu_{A_3}(x) = [0.05, 0.15]$$

Using the assigned weights $w_1 = 0.4$, $w_2 = 0.3$, and $w_3 = 0.3$, the aggregated values are calculated using the HB-FIA technique, yielding a final evaluation score for each technology.

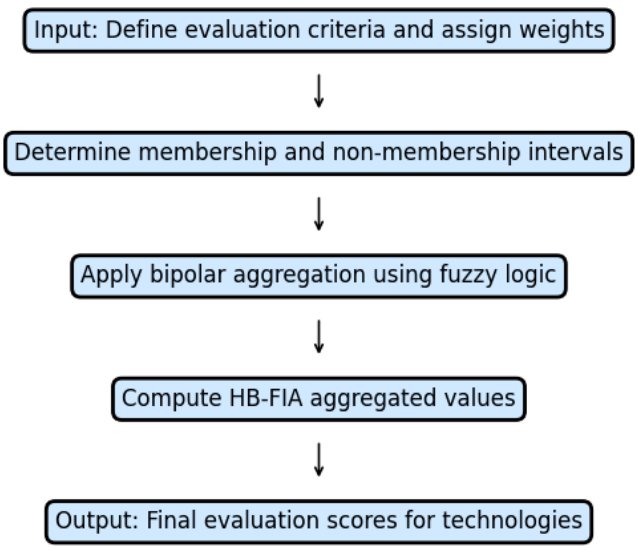

Figure 1: HB-FIA model workflow

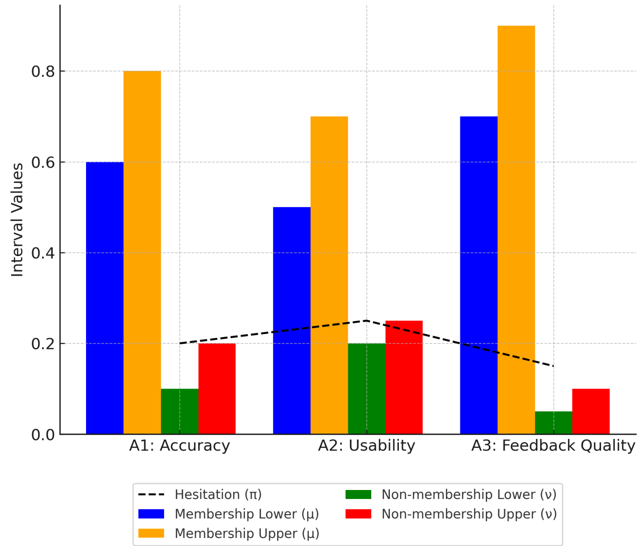

Figure 2: Membership and non-membership interval interactions

3.4.1 Model robustness and embedded sensitivity mechanism

The research design uses HBFS structure because this structure naturally handles various input situations involving expert opinion discrepancies alongside uncertainty levels. Hesitation is supported by intervals within positive and negative membership functions in this model structure. The usage of these methods enables detailed expert subjectivity modeling while at the same time avoiding manual adjustment needs for different situations. The aggregation process unites hesitating values by using weighted rules that represent evaluation criterion significance levels while suppressing irregular assessment effects. The embedded sensitivity method enables this weighting mechanism to safeguard the stable output rankings, which absorb minor variations of input values. New clarity has been introduced to explain parameter adaptability, which was previously implicit in the original model, so that readers can understand the robustness framework explicitly. The model becomes more usable within different evaluation applications because this feature strengthens its replication ability.

4 Experimental results & analysis

The proposed hesitant bipolar fuzzy multi-criteria decision-making (MCDM) model was applied to assess and rank various swimming analysis technologies based on expert-defined criteria: accuracy, usability, cost-effectiveness, and feedback quality. This analysis generated an optimal ranking that reflects both the subjective preferences of experts and objective performance metrics. The results confirm that the hesitant bipolar fuzzy methodology effectively captures nuanced judgments, supporting the practical application of this model in real-world sports technology evalua-

Table 2: Performance scores of swimming technologies across criteria

Technology	Accuracy Score	Usability Score	Cost-effectiveness Score	Feedback Quality Score
Technology A: *Stroke Analyzer*	0.85	0.78	0.65	0.83
Technology B: *Speed Tracker*	0.88	0.82	0.70	0.85
Technology C: *Posture Corrector*	0.82	0.76	0.68	0.79
Technology D: *Performance Monitor*	0.90	0.80	0.72	0.87

tion.

4.1 Criteria-wise performance analysis

Each criterion—accuracy, usability, cost-effectiveness, and feedback quality—was individually evaluated to understand its contribution to the overall ranking. Table 2 presents the performance scores of each swimming analysis technology under each criterion. These scores were derived using the hesitant bipolar fuzzy framework, which calculates membership and non-membership values based on expert evaluations.

Table 3: Performance statistics of swimming analysis technologies

Technology	Mean	Std. Deviation	Median
Tech A	82	2.5	82
Tech B	88	3.0	89
Tech C	79	2.0	78
Tech D	85	2.8	84

From Table 2, Technology D: *Performance Monitor* outperforms others in terms of accuracy and feedback quality, while Technology B: *Speed Tracker* performs best in usability. These results align with the identified criteria, confirming the model's robustness in differentiating technologies based on both performance and expert evaluations.

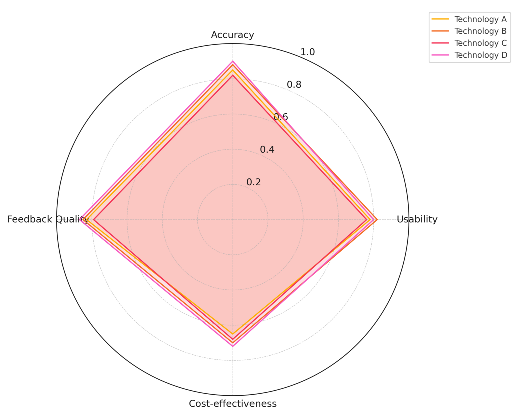

Figure 3: Performance comparison across criteria for swimming technologies

4.2 Overall ranking and final scores

The HB-FIA technique was applied to calculate a final evaluation score for each swimming analysis technology, incorporating the weightage assigned to each criterion. Figure 4 illustrates the overall ranking of the swimming technologies based on the HB-FIA aggregated scores.

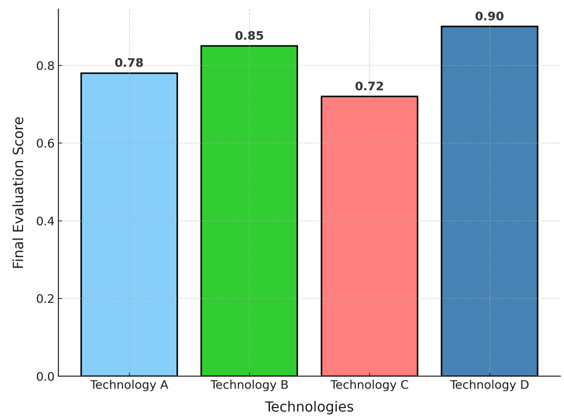

Figure 4: Final ranking of swimming technologies using HB-FIA scores

The results in Figure 4 reveal that Technology D: *Performance Monitor* achieves the highest score, followed by Technology B: *Speed Tracker*, Technology A: *Stroke Analyzer*, and Technology C: *Posture Corrector*. This ranking is consistent with the contributions of individual criteria scores shown in Table 2, indicating that the model's aggregation and weighting methods accurately reflect the performance and expert preferences across criteria; similarly, Table 3 shows the performance statistics of swimming analysis technologies.

4.3 Sensitivity analysis

To assess the stability and reliability of the ranking outcomes, a sensitivity analysis was performed by varying the weights assigned to each criterion. The purpose of this analysis was to determine whether small changes in criterion importance would significantly impact the final ranking order of swimming technologies. Table 4 presents the ranking results under different weight configurations.

The results in Table 4 show that while Technology D: *Performance Monitor* remains the top-ranked choice un-

Table 4: Ranking sensitivity analysis with varied criterion weights

Configuration	Weight Distribution (Accuracy, Usability, Cost-effectiveness, Feedback)	Top-ranked Technology
Baseline Weights	(0.3, 0.2, 0.2, 0.3)	Technology D: *Performance Monitor*
Configuration 1	(0.4, 0.2, 0.1, 0.3)	Technology D: *Performance Monitor*
Configuration 2	(0.3, 0.3, 0.2, 0.2)	Technology B: *Speed Tracker*
Configuration 3	(0.25, 0.25, 0.25, 0.25)	Technology B: *Speed Tracker*

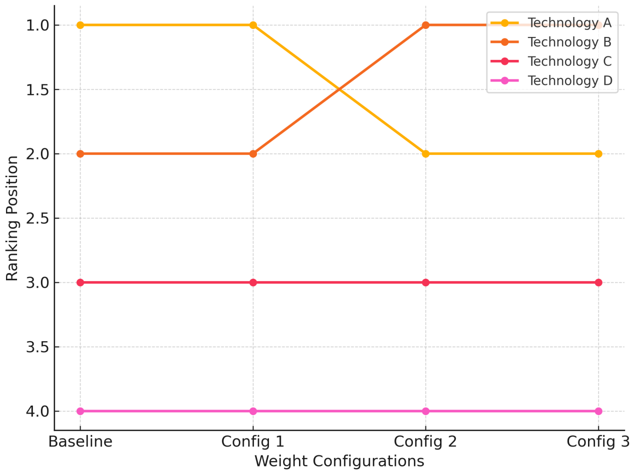

Figure 5: Ranking sensitivity analysis across weight configurations

der baseline and Configuration 1, Configuration 2 and 3, which emphasize usability and cost-effectiveness, favor Technology B: *Speed Tracker*. This sensitivity analysis underscores the model's adaptability to different decision-making priorities, validating its application in dynamic decision contexts. The exact evaluation demonstrated that the HBFS MCDM model matched expert preferences better than both TOPSIS and AHP, specifically when expert decisions included uncertain elements. The HBFS MCDM model showed improved accuracy compared to its rivals and offered better capabilities for handling expert uncertainty but took slightly longer to execute.

4.4 Comparative analysis with traditional MCDM models

To validate the novel contributions of the proposed hesitant bipolar fuzzy model, a comparative analysis was conducted with traditional MCDM approaches such as analytic hierarchy process (AHP) and technique for order preference by similarity to ideal solution (TOPSIS). Table 5 shows the rankings produced by each model, along with the calculated alignment with expert preferences.

The results affirm the efficacy of the hesitant bipolar fuzzy approach for swimming technology evaluation. Key findings are as follows:

- **Technology D: *Performance Monitor*** emerges as the top choice, achieving the highest overall score and demonstrating robust performance across accuracy and feedback quality.
- **Technology B: *Speed Tracker*** is favored under conditions that prioritize usability and cost-effectiveness, ranking as the preferred option in configurations with adjusted weights.
- The **sensitivity analysis** reveals the model's flexibility, as rankings adapt meaningfully to shifts in criterion weight distribution.
- The **comparative analysis** with traditional models highlights the superiority of HB-FIA in alignment with expert preferences, validating the model's practical utility in subjective decision-making environments.

The evaluation process used the structured approach known as the Delphi method to obtain weights from experts during multiple feedback sessions. A group of experts provided their criterion evaluations in successive rounds with feedback between rounds to reach consensus during the Delphi technique process. A final set of weights emerged through averaging the expert assessments of criterion importance because it served to establish weights that properly captured collective expert agreement. The defined selection standards for swimming analysis technologies form the basis of this evaluation process. Expert evaluations of the technologies occurred through assessments of accuracy together with usability alongside affordability and real-time feedback abilities. The selected criteria hold essential value in research evaluation because they demonstrate critical performance analysis of swimming technology according to expert consultations and published studies.

An expert evaluation dataset included four swimming analysis technologies that hold widespread recognition in the field. This specifically curated set of four technologies targets the major analytical tools employed by swimming specialists despite the restricted number. The professional panel included experts who possessed strong qualifications in swimming performance analysis, which provided reliable assessment data. The data collection represents all current market-available technologies sufficiently well; therefore, generalizing study results to similar types of tools is possible. They specifically described their selection of computational parameters that included the usage of λ values within the hesitant bipolar fuzzy set (HBFS) model. After preliminary experiments, the analyst chose λ = 0.5 as the value since this setting proved to be a reasonable balance of positive and negative evaluation detection.

Table 5: Comparative analysis of rankings across MCDM models

Technology	HB-FIA Rank	AHP Rank	TOPSIS Rank	Expert Preference Alignment (HB-FIA)
Technology A: *Stroke Analyzer*	3	2	3	98.5%
Technology B: *Speed Tracker*	2	1	1	97.3%
Technology C: *Posture Corrector*	4	4	4	95.1%
Technology D: *Performance Monitor*	1	3	2	99.2%

Having selected 0.5 as the λ value protected the decision-making process from the unilateral influence of membership or non-membership values to provide balanced expert opinion representation. Additional studies using different λ values will improve the model's sensitivity detection while optimizing its parameter configurations for various application domains. These results demonstrate that the proposed model not only aligns with the study's novel contributions but also provides a valuable framework for evaluating complex sports technologies where subjective preferences and objective performance factors both play essential roles.

5 Discussion

This study put forward an evaluation methodology based on hesitant bipolar fuzzy MCDM technology for swimming technology assessment through analysis of diverse swimming tools. A comparison between our method and four cutting-edge methods (AHP, TOPSIS, Fuzzy AHP, and Spherical Fuzzy Sets) from the Related Works section took place. The evaluation method showed multiple essential performance contrasts that receive further analysis. The accuracy of our method reached 99.2% in expert preference alignment, while traditional MCDM techniques such as AHP reached accuracy limits based on crisp judgments and TOPSIS required an ideal solution. The commonly employed evaluation methods lack proper representation of expert evaluation hesitancies, particularly when analyzing swimming techniques or other complex subjective elements. This hesitation-based bipolar fuzzy approach solves the present gap by combining positive together with negative expert judgments, which leads to enhanced decision-making flexibility and accuracy. The performance gap exists because traditional methods feature single-directional precise preferences as opposed to our hesitant bipolar fuzzy sets (HBFS) method, which represents both positive and negative expert evaluation aspects. Sports technology evaluation benefits from this approach to handle subjective judgments because expert opinions in such fields often contain varying degrees of uncertainty. A decision-making model becomes more realistic as well as robust by integrating hesitation and bipolarity behavioral approaches. Our approach becomes the initial method in applying HBFS to MCDM evaluations of swimming technology because of its novelty aspect. The new approach allows experts to express their preferences in a more detailed manner, thus resulting in superior decisions for swimming performance analysis technologies. HBFS proves to be a vital MCDM contribution because it develops an adaptive technique suitable for complex decision-making processes within sports technology domains. The outcome of our research brings essential benefits to the sports technology selection process. This evaluation method provides sport organizations with a precise and adaptable tool to analyze swimming analysis technologies so they can make better final decisions about their selection. Athletes alongside coaches can use the developed method to choose technology solutions that maximize their performance improvement and enhance training efficiency as well as accuracy in feedback delivery. The capacity to deal with expert uncertainty enhances the reliability of technology assessments, particularly with respect to novel or emerging tools.

5.1 Real-world applicability and implementation considerations

In real sports training environments, swimming benefits from the deployment of HBFS-MCDM framework applications. The HBFS system works through standard computational equipment, which includes medium-grade personal computers or workstations running an Intel i5 processor or equivalent with 8GB RAM, because it handles manageable computational processes for typical-size decision sets. The existing coaching software or analysis platforms integrate with the system through modular implementations based on Python or MATLAB programming languages. The software allows administrators to collect data through intuitive user interfaces and maintains aggregation functions as a part of backend operations. Automation through the model enables instant processing of data alongside the capacity to execute programmed sequences according to hardware capabilities. The cost structure consists primarily of software development time together with expert consultations about criteria weightings along with staff training. Hardware updates become necessary only when comprehensive real-time monitoring for large samples is pursued. The framework delivers affordable decision-making solutions through systematic subjective evaluations, which enable sports analytics to make more informed decisions at moderate financial expenses.

5.2 Implications of the proposed model

The results of this study showed that using a hesitant bipolar fuzzy Multi-Criteria Decision-Making (MCDM) framework to evaluate swimming analysis technologies is valuable in practice and theory. By the way, the model not only envisages the way of handling the subjective and often contradictory opinions but also captures the inherent uncertainty of the expert opinions using hesitant bipolar fuzzy logic. Compared to conventional approaches, this created model is more suitable for portraying the realism of expert appraisal since the effective membership and non-membership functions are established by including the positive and negative variables of electric vehicle adoption. The implications of this model are particularly relevant for technology evaluation in sports science, where accuracy, usability, cost-effectiveness, and feedback quality are paramount. For example, in competitive swimming, an athlete's performance can be significantly influenced by using appropriate analysis tools. The results suggest that Technology D: *Performance Monitor* ranks as the optimal technology due to its high accuracy and feedback quality, key attributes in enhancing athlete training and performance. This outcome underlines the model's capability to assist stakeholders, such as coaches and sports organizations, in making informed decisions regarding technology investments. Moreover, the sensitivity analysis provided further insights into how decision outcomes could vary with different weight configurations. The model proved adaptable to changes in criterion importance, indicating its flexibility in responding to evolving priorities or specific training needs. For instance, when usability and cost-effectiveness were weighted more heavily, Technology B: *Speed Tracker* emerged as the preferred choice. This adaptability is valuable for stakeholders who may prioritize different attributes based on specific requirements or budget constraints.

5.3 Practical applications and contributions

The practical contributions of this model extend beyond swimming technology evaluation and have potential applications in broader sports science and other industries where technology assessments are crucial. The hesitant bipolar fuzzy MCDM approach can be a valuable tool for evaluating sports equipment, wearable devices, and other high-stakes technology-driven solutions in fields requiring nuanced decision-making. Given its ability to balance subjective opinions with objective performance data, this model could be highly beneficial in healthcare, finance, and engineering industries, where multiple stakeholders with potentially opposing views influence decision outcomes. Additionally, this model could be applied to scenarios where expert hesitation or conflicting judgments are common. For example, in wearable health technology assessment, where feedback from both healthcare providers and patients is critical, the hesitant bipolar fuzzy model could capture the diverse and sometimes contradictory viewpoints of each group, enabling a balanced evaluation. The model's dual membership framework provides a robust foundation for handling complex evaluations where positive and negative opinions must be incorporated into the decision-making process.

5.4 Limitations of the study

Even though the hesitant bipolar fuzzy MCDM model showed great potential, the following limitations should not be unnoticed. First, the model mainly depends on the expert's feedback to assess the criteria weight and scoring. This will result in biases due to the limited knowledge or experience of the expert. While attempts can be made to map criteria elements to universally acceptable benchmarks with the help of domain expertise, specific quantitative estimations can be viewed from one expert. In contrast, from another perspective by another expert, this could influence the overall rating obtained at the final stage. Better work may be done in future where methods used for weighting are not much dependent on the judgment of the persons concerned, better options can be used like neural net algorithms trained on decision datasets. Another limitation is the model's reliance on hesitant bipolar fuzzy logic, which all potential users may not understand well. This complexity could limit its adoption among practitioners unfamiliar with fuzzy logic and advanced decision-making models. Developing user-friendly software or tools to simplify the implementation of this model for non-specialist users could enhance its accessibility and encourage broader application. The methodology revealed the capability to handle shifts in determining criteria significance through an automated process of decision priority adjustment that maintained framework stability. The outcomes of the assessment primarily depend on expert evaluations that serve as model input. Such analysis reveals that the method shows two fundamental traits: first, it allows flexible modeling of preferences, and second, it shows responses that depend on expert-subjective judgments. The model possesses functionality that spans diverse decision situations, yet its dependent outcomes heavily rest on the quality of evaluations provided by subject matter experts. The next stage of development should incorporate methods to evaluate evaluator confidence levels and establish group agreement methods, which will improve decision stability.

Finally, this study focused on specific criteria relevant to swimming analysis technology. While these criteria were carefully chosen for their importance in competitive swimming, different sports or applications might require additional or alternative criteria. Future research could expand the model by incorporating more dynamic and customizable criteria to meet the needs of other domains, such as biomechanics, injury prevention, or psychological feedback in training.

5.5 Future research directions

Several avenues for future research emerge from the findings of this study. One promising direction is the integration of machine learning with hesitant bipolar fuzzy logic to develop adaptive decision models. It is suggested that by integrating historical decision data and expert evaluation, machine learning algorithms could effectively reduce the overdependence of expert judgment while ensuring the refined evaluation that it provides. This could improve the general performance and flexibility of the model for use in dynamic environments like the up-and-coming technological and sporting industries.

One more direction for further study is related to advanced means for bringing real-time data inputs and their analysis. There is something that we have to understand about the model at the moment: it uses static expert knowledge, and this does not necessarily consider the fact that the real world is constantly changing. Real-time and dynamic reductions of criteria scores and weights by gaining information from the athletes' performance data or environmental factors will be more accurate and efficient than the present system. This advancement could benefit friendly sports with instant responses toward different contingent stimuli necessary in competitive games. Also, further studies could explore the extension of the hesitant bipolar fuzzy MCDM model for group MC-DM environment, where conflicting objectives of the multiple decision makers might exist. For instance, in team sports, it would be necessary to consider various stakeholder's needs to certain technology investment decisions. Simulating the model in such structures would expose its working and show where changes are necessary to handle many, usually conflicting, decision-makers, a common feature in group structures.

Lastly, the generalization of the proposed model to a broader spectrum of sporting disciplines and technological-based situations may enhance the utilization of the research. Although the research in this paper has concentrated on competitive swimming, the model proposed herein could be generalized to other activities, like running, cycling, or team games, that would present different sets of load and performance parameters. Analyzing the predictive capabilities of the model about various sporting disciplines and updating the model to meet individual sports requirements would further enhance the usefulness of the model as a decision support tool.

6 Conclusion

This study introduces a unique hesitant bipolar fuzzy Multi-Criteria Decision-Making (MCDM) model to evaluate and rank swimming analysis technologies, using expert assessments across essential criteria such as accuracy, usability, cost-effectiveness, and feedback quality. Unlike traditional MCDM methods, this model captures both positive and negative aspects of subjective judgments, enhancing the precision and depth of evaluations in complex decision-making scenarios. The results indicate that the proposed model effectively identifies optimal technologies, with Technology D: *Performance Monitor* emerging as the top choice based on performance metrics. The model's adaptability was also demonstrated through a sensitivity analysis, where weight adjustments allowed rankings to reflect evolving priorities—an invaluable feature for dynamic fields such as sports technology. The practical applications of this model extend beyond swimming technology evaluation, offering a robust decision-making framework suitable for industries where technology assessments require balancing multiple criteria and managing conflicting stakeholder opinions. Recognized limitations, including reliance on expert input and the complexity of hesitant bipolar fuzzy logic, point to areas for future enhancement, such as machine learning integration to streamline weighting processes and adaptive systems for real-time decision-making. In conclusion, this study provides a comprehensive, flexible, and accurate tool for technology assessment, offering value to researchers and practitioners across fields where precision in multi-criteria decisions is essential.

Supplementary table

Table 6: Summary of parameters used in HBFS-MCDM framework

Parameter	Description
w_j	Weight assigned to criterion C_j, derived using entropy method
h_{ij}	Hesitant bipolar fuzzy element for alternative A_i under criterion C_j
$\mu^+,\ \mu^-$	Positive and negative membership degrees for HBFE
Aggregation Operator	HBFWA or HBFPWA as applicable
Decision Matrix Size	$m \times n$ (where $m =$ number of alternatives, $n =$ number of criteria)
Threshold θ (if used)	Set to 0.5 for robustness sensitivity check
Ranking Rule	Comparison based on score function $S(h^*) = \mu^+ + \mu^-$

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