

Event-Triggered Predictive Control Algorithm for Multi-AUV Formation Modeling

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Autonomous underwater robot has high flexibility and autonomy. Multi-autonomous underwater robot formation is its main research direction. In order to improve the application effect of this formation control method, event triggering mechanism and model predictive control are introduced in the experiment for method design. At the same time, neural network and filtering control are introduced in the experiment for the optimization of the method. The autonomous underwater robots 2A~5A were able to smoothly avoid the obstacles under the leader 1A, as demonstrated by the trial results. Autonomous underwater robots 1~5 had a maximum error of 3.8 m and a maximum velocity error of 3.7 m/s. After a period of time, their position and velocity errors converged to 0. The proposed method had a maximum rms of 2.4233 and an average rms of 1.4015. It required the least number of triggers of all the methods for the optimization problem solution. The above results confirm that the multi-autonomous underwater robot formation control method based on event-triggered mechanism and model predictive control proposed in the study can realize efficient and accurate control, and can reduce the difficulty of computation and resource consumption.

Povzetek: Algoritem za vodenje formacije več avtonomnih podvodnih vozil uporablja mehanizem sproženja dogodkov in model napovedne regulacije, optimiziran z nevronskimi mrežami

1 Introduction

Autonomous underwater vehicle (AUV) has high flexibility and autonomy, and has been well applied in the fields of environmental detection and underwater rescue [1]. However, the carrying space of a single AUV is limited and cannot adapt to some complex scenes. Therefore, multi-AUV collaboration mode is considered to work in practical applications. Multiple AUV collaboration can improve the efficiency, communication capability and flexibility of operation, which requires formation cooperative control. However, the existing methods have poor formation stabilization ability or are difficult to avoid obstacles, etc., so they also need to be improved. The operation system of AUVs involves the design of controllers, which belongs to the core part of AUVs. Adaptive control, neural network (NN) control and proportional-integral-derivative controllers (PID) are commonly used control methods [2]. However, these methods are computationally difficult and some of them can only be applied to certain fixed scenarios. For example, although the NN-based control method can realize more accurate control, it is computationally intensive and less effective in real-time control. Therefore, it is necessary to consider a suitable method for multi-AUV cooperative scenarios in the controller design of AUVs. Model predictive control (MPC) belongs to a closed-loop optimal control strategy. It shows strong robustness and has better control effect, so it can be used in path optimization of intelligent devices such as

unmanned aerial vehicles [3]. However, it has a large amount of computation, so it is somewhat limited in practical applications. Event triggering mechanism (ETM) can reduce the computation of the control model [4]. Therefore, this work takes into account the combination of MPC and ETM in order to increase the efficiency and stability of multi-autonomous underwater vehicle formation (MAUVF) control. This is the innovation of the article. The article consists of five chapters. Firstly, the research of others is summarized. Secondly, the MAUVF control and its optimization based on MPC and ETM are described. The third section is the performance test of the method and the analysis of the application effect. The fourth section is a discussion and analysis of the results of the paper. Finally, the article is summarized as well as the outlook. It is hoped that the improvement of the MAUVF control method can improve the efficiency and safety of AUVs underwater.

2 Related works

AUV detection in the ocean needs to consider the accuracy and efficiency of information transmission. Liu et al. proposed to optimize the transmission scheduling of AUVs using an improved genetic algorithm and updated the communication network. After simulation and analysis, this method was able to improve the signal quality in the AUV formation and achieve efficient information exchange [5]. The formation of AUVs involved communication problems and needed to adjust the related parameters. Cao et al. utilized methods such as

factorial design to estimate measurement parameters such as signal-to-noise ratio. At the same time, they performed AUV formation based on the construction prediction method. This method can effectively avoid local optimization and reduce the dependence on parameters [6]. If multiple AUVs are to establish an appropriate obstacle avoidance strategy, then a reasonable planning path is needed. A variety of techniques, including an enhanced artificial potential field, were employed by Pang et al. for route planning and navigation. This method enabled multiple AUVs to realize real-time formation and pass-through obstacles smoothly according to the environment [7]. Yu et al. introduced some constraints in the formation control of AUVs for improving the accuracy of communication. They introduced different localization methods in the model and discussed the technical difficulties in applying acoustic and optical communication in the control model [8]. These studies provide a reference for realizing intelligent AUV formation.

Optimal control problems can be resolved with the help of MPC, a unique control technique. In the study of Angelov et al., the accuracy of the prediction results of MPC in open-loop control problems was low. To improve the solving ability of MPC in optimal control problems, it also needs to be improved by combining it with other methods. They confirmed that the improved MPC can accurately estimate the error in the open-loop problem and realize accurate prediction [9]. In the study of obstacle avoidance in robots, MPC was able to combine with other methods for reasonable path planning. Zhou et al. combined MPC with randomized tree algorithm to achieve path perception for robots. MPC improved the safe distance between the robot and obstacles. In simulation experiments, this method helped the robot to avoid obstacles successfully [10]. MPC can also be used in path optimization for AUVs. Liu et al. used MPC to design a path planning algorithm capable of avoiding obstacles. This method was able to generate path points and satisfy the set constraints. In real-world path planning, this method can enable the AUV to successfully avoid obstacles and obtain an optimal path [11]. Gong et al. designed a control model related to AUV trajectories using MPC. They incorporated backstepping control in the model and enabled convergence of the AUV trajectory. This approach improved the robustness and stability of the model, thus improving the tracking capability of the AUV [12]. Bian et al. designed a model

that enables coordinated diving of multiple AUVs using distributed MPC. In this approach, they determined the stability conditions for multi-AUV diving and saved reduced communication costs. The stability, efficiency and economy of this approach were confirmed in simulation experiments [13].

The combination of ETM and NN can improve the security of communication. Zhang F et al. designed a security control model based on ETM. They introduced adaptive NN in this model for improving the time stability in closed-loop systems. Experiments confirmed the safety and effectiveness of this model [14]. In formation control of multiple AUVs, ETM can reduce ineffective communication and thus improve communication efficiency. According to Li, increasing the error threshold computation in ETM can boost the formation control model's computational effectiveness. Meanwhile, the introduction of Hungarian algorithm can solve the problem of model check-in failure. In simulation experiments, this method can significantly reduce the communication traffic and can perform accurate guidance [15]. Dynamic event triggering (DET) is capable of balancing communication frequencies in formation control of underwater multi-AUVs. Su et al. developed an error estimation model based on DET while introducing an adaptive approach. Simulation experiments confirmed the real-time and stability of this method [16]. Wang et al. used the artificial potential field approach with ETM and other factors to build a formation control model for numerous AUVs. Meanwhile, they introduced a fixed-time trigger mechanism in this model for communication efficiency optimization. In underwater simulation experiments, this method can effectively reduce the communication energy consumption but does not affect the quality of communication [17].

The above studies show that MPC and ETM exhibit better application in formation control of AUVs. Both of them can guarantee the stability of the control model and lessen communication loss in the formation. However, there are fewer studies on the joint application of the two in MAUVF control. Therefore, it is thought that the combination of MPC and ETM will be utilized in MAUVF control in the experiment in order to further increase the model's efficiency and decrease the loss. Table 1 summarizes the main work and shortcomings of the references.

Table 1: Summary of references

Reference	Key performance indicators	Limitations
[5]	Improved signal quality in AUV formation and achieved efficient information exchange	Insufficient detection capability in low-quality underwater acoustic channel detection
[6]	Avoiding local optima and reducing dependence on parameters	In the case of fading channels, the stability of the model is poor
[7]	Enable multiple AUVs to achieve real-time formation and smoothly pass-through obstacles based on their environment	Poor formation control of formation AUVs when passing through obstacle zones

[8]	Introduced some constraints to improve communication accuracy	The real-time control effect of the method is poor
[9]	Accurately estimating errors in open-loop problems	The application scope of the method is limited
[10]	Improved safety distance between robots and obstacles	The application scope of the method is limited
[11]	Capable of generating path points and meeting set constraints	The method requires a large amount of computation
[12]	Combining backstepping control and ensuring the convergence of AUV operation trajectory	The model exhibits dynamic uncertainty and is susceptible to external disturbances from ocean currents
[13]	Confirmed stability conditions for multi AUV diving and reduced communication costs	Long triggering interval increases the computational burden of the model
[14]	Proved safety and effectiveness	Stability needs to be improved
[15]	Significantly reduce communication traffic	Low computational efficiency
[16]	Strong real-time and stability	The accuracy of error estimation needs improvement
[17]	Effectively reducing communication energy consumption	Communication efficiency is greatly affected by external factors

3 Multi-AUV formation approach based on event-triggered and model-controlled prediction algorithms

One major element influencing how well AUVs operate is the MAUVF control technique. As a result, it's essential to create a control strategy that's both practical and effective. In the first subsection, an MAUVF control model is developed using a combination of MPC and ETM. Then in the second subsection, improvements are made to address the shortcomings of this model in order to improve the practical use of this formation control method.

3.1 Control methods for multi-AUV formations

MPC is a component of the closed-loop optimal control strategy, which solves the open-loop optimum control issue to arrive at the current control action [18]. It exhibits strong robustness and has good control effect, so it can be used in path optimization of intelligent devices such as UAVs. A mathematical model is required to describe the overall changes in the system when using MPC in the MAUVF control approach [19]. It allows the control of input information based on current system information and future predictions to predict future output information. In Figure 1 the schematic diagram of MPC is shown.

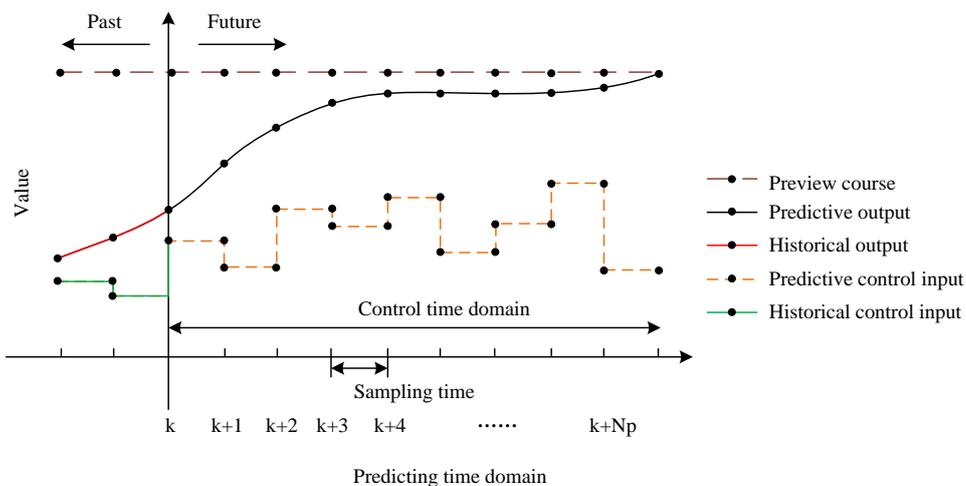


Figure 1: Schematic diagram of model predictive control

In Figure 1, there is a desired trajectory in the MPC as a reference. Taking the time as a starting point, the system

performs feedback control through the established prediction model and the current state of the platform

during the $[k, k + N_p]$ time period. This solves for the predicted output values. By solving the objective function under various constraints, one can obtain the actual inputs in the control sequence. Repeating these steps will lead to the realization of MPC. When applying it to MAUVFs, it is necessary to develop suitable mathematical models for analyzing the trajectories of the AUVs, among other factors.

To realize the MAUVF control, the pilot-follower formation method is adopted in the experiment for cooperative control. However, it is easy to encounter obstacles such as reefs and buoys in actual navigation, so the specific environment also needs to be analyzed. Analyzing the force acting on the AUV in the coordinate system is vital for the MAUVF control to increase the efficacy of the formation movement control. A schematic depiction of the AUV's movement in three different coordinate system directions is displayed in Figure 2.

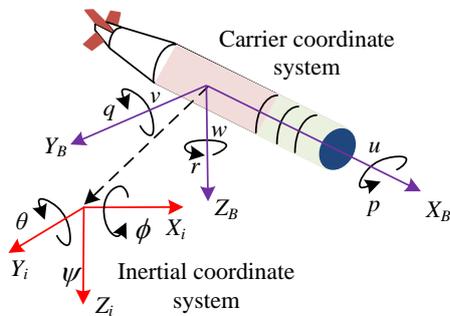


Figure 2: Schematic diagram of coordinate relationship during AUV motion

In Figure 2, i represents the number of AUVs. v represents the velocity information. q represents the longitudinal angular velocity. u represents longitudinal velocity. w represents vertical velocity. p represents position information. ψ represents heading angle. ϕ represents roll angle. X_i, Y_i, Z_i is the inertial coordinate system in three directions. X_B, Y_B, Z_B is the three directions carrier coordinate system. The AUV will move in and out, pan and float along the three different directions of X_B, Y_B, Z_B . If the MAUVF control system has a lead AUV and other following AUVs, each following AUV is equipped with sensors to receive position and orientation relative to neighboring AUVs. It can then use the inertial coordinate system for its own position and orientation determination. A schematic depiction of the pilot-follow formation with AUV1 as the pilot is shown in Figure 3.

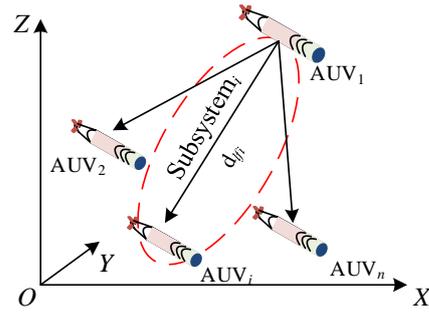


Figure 3: Schematic diagram of navigation-following formation

In Figure 3, the action trajectory of AUV1 is pre-set. d_{fi} denotes the relatively safer desired distance between AUV1 and the following AUVi in formation control. The following AUVs follow AUV1 within a certain distance based on a controller combining MPC and ETM. The time needed to make the stabilizing signal arrive in this process. Therefore, equation (1) can be used to express the primary goal of MAUVF regulation.

$$\begin{cases} \lim_{t \rightarrow T} \|p_1(t) - p_i(t) - d_{fi}\| = 0 \\ \lim_{t \rightarrow T} \|v_1(t) - v_i(t)\| = 0 \end{cases} \quad (1)$$

In equation (1), ϕ denotes the convergence time. Equation (2) expresses it in relation to the formation control parameters, irrespective of the system's initial state.

$$T \leq T_{\max} = \frac{1}{\gamma_1(1-\alpha)} + \frac{1}{\gamma_2(\beta-1)} \quad (2)$$

In equation (2), $\gamma_1, \gamma_2, \alpha, \beta$ is a constant. Equation (3) is a descriptive method for following AUVi.

$$\begin{cases} p_i^g = J(\Theta_i)v_i \\ M_i^g v_i = -D(v_i)v_i - g(\Theta_i) + \tau_i \end{cases} \quad (3)$$

In equation (3), M is the inertia matrix. Θ represents the attitude quantity. τ is the control input force moment. D represent the damping matrix. $g(\Theta_i)$ denotes the restoring force matrix. The experiment uses the inverse step approach with dynamic surface algorithm to construct the control algorithm based on the preceding formulas. This reduces the computational difficulty and avoids computational explosion. Fixed time and distributed ETM are introduced in this method. The MPC is triggered when the state error between the following AUV and the leader AUV1 satisfies the trigger condition. This can improve the convergence speed of MAUVF while reducing the energy consumption of the system. The primary purpose of the MPC controller for MAUVF

techniques is to minimize the discrepancy between the AUVs' operational route and reference route. This problem can be transformed into optimizing the constraints of states and inputs. The optimal inputs to the controller can be found in the experiments based on the current state the robot is in and the expected trajectory. The AUVs are then able to perform trajectory tracking based on the optimal inputs, ultimately realizing MAUVF control.

The experiment utilizes the MPC model for feedback correction to overcome environmental interference and maintain closed-loop stability. Predictive models are often nonlinear and subject to unstable factors, such as disturbances and time-varying variations. This can result in a mismatch between the established predictive model and the actual controlled object, rendering MPC unable to accurately match the actual control process. Thus, a feedback loop is incorporated into the control process. At the start of each sampling interval, the output of the controlled object is detected, and the model's predicted results are adjusted based on the detection results. Subsequently, new optimizations are performed to achieve the desired control effect.

Although the above MPC-based MAUVF control method can get better control effect, the computational amount of this method is relatively large. At the same time, it will reduce the accuracy of formation control when facing the underwater complex environment. Therefore, the method needs to be further improved in order to enhance the practical application of the method. In the next section, this formation control method is improved.

3.2 Multi-AUV formation method based on improved event triggering mechanism

Because of the uncertainty of the AUV model and the complexity of the underwater environment, MPC is more challenging to implement accurately in formation control. With its large capacity for nonlinear processing, NN can be applied to solve the MAUVF control non-deficiency problem. Radial basis function neural network (RBFNN) is introduced in the experiment to increase the system's robustness. It has strong learning ability and can approximate the nonlinear terms, equation (4) is the input and output of RBFNN.

$$\begin{cases} S_r(Z_r) = \exp(-\frac{\|Z_r - o_r\|}{2\sigma_r^2}) \\ f(Z) = W^T S(Z) + \delta(Z) \end{cases} \quad (4)$$

In equation (4), S denotes the output vector. Z denotes the input of RBFNN. o is the center vector of node i . W denotes the ideal weight matrix. δ denotes the approximation error. σ denotes the width parameter. In practical applications, the input and output moments of the AUV are limited because of physical conditions. However, the actuator saturation of the AUV is not taken into account in most of the control algorithms. Therefore, based on the previous section, the effect of the input saturation situation on the MPC formation is considered in the experiments. The improved MAUVF control method is depicted in Figure 4.

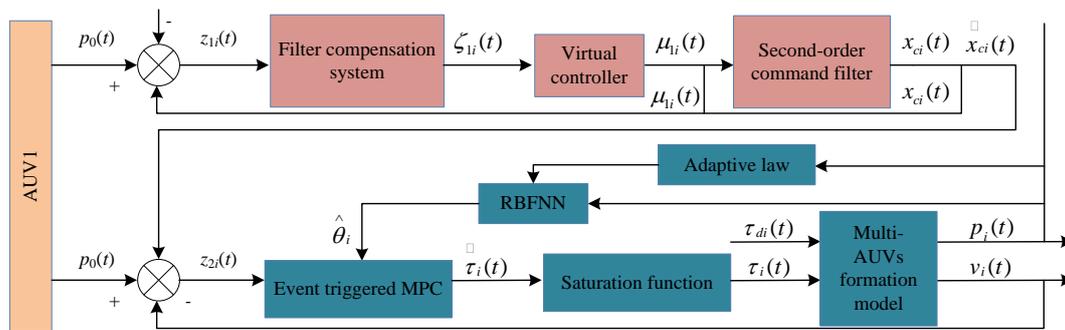


Figure 4: Improved multi-AUVs formation control method

In Figure 4, τ denotes the control input. τ_{di} denotes the unknown external perturbation. z denotes the error. Equation (5) shows the mathematical expression of the improved formation control method.

$$\begin{cases} \dot{p}_i(t) = J(\Theta_i)v_i(t) \\ M_i \dot{v}_i(t) = -D(v_i(t))v_i(t) - g(\Theta_i) + \tau_i(t) + \tau_{di}(t) \end{cases} \quad (5)$$

$x_{1i}(t) = p_i(t)$ and $x_{2i}(t) = \dot{p}_i(t)$ are defined in the experiment, then equation (6) can be obtained.

$$\begin{cases} \dot{x}_{1i}(t) = x_{2i}(t) \\ \dot{x}_{2i}(t) = f_i(\Theta_i, v(t)) + \tau_i(t) \end{cases} \quad (6)$$

In equation (6), $\tau_i(t) = JM_i^{-1}\tau_i(t)$, it denotes a nonlinear control input with input saturation characteristics. From the saturation characteristic, it is known that the input signal will change abruptly when certain conditions are

satisfied. Meanwhile, the functions involved in the backstepping method and the designed formation control method are smooth. In order to approximate the saturation function, a smooth function must also be created; in the experiment, the tanh hyperbolic tangent function is used. This function improves the ability of the formation control method to cope with unknown perturbations under input saturation constraints. To achieve the control objective stated in equation (1), the experiment utilizes a combination of filtered control and fixed-time methods resulting in the improved event-triggered formation-based control method shown in Figure 4. This method achieves a steady state with a fixed time and is not affected by the initial state of the formation model, thus avoiding Zeno behavior. Equation (7) shows the position error of the formation control method.

$$z_{1i}(t) = x_{1i}(t) - x_d(t) - x_{ai} \quad (7)$$

In Equation (7), x_{ai} denotes the safe distance between AUV1 and the follower. x_d denotes the desired position signal. The combination of second order command filter and backstepping is carried out in the improved method, which reduces the difficulty of designing the MPC controller. Equation (8) shows the calculation of this filter.

$$\begin{cases} \overset{g}{\phi}_{1i}(t) = t\overset{g}{\phi}_{2i}(t) \\ \overset{g}{\phi}_{2i}(t) = -2\kappa t - t(\overset{g}{\phi}_{1i}(t) - \mu_{1i}(t)) \end{cases} \quad (8)$$

In equation (8), κ, t denotes the constant. $\mu_{1i}(t)$ denotes the input signal of the filter. Equation (9) shows the velocity error of the improved formation method.

$$z_{2i}(t) = x_{2i}(t) - x_{ci}(t) \quad (9)$$

According to the calculation in equation (8), a filtered error is given in equation (10).

$$\mathbf{V}\mu_{1i}(t) = x_{ci}(t) - \mu_{1i}(t) \quad (10)$$

The error in equation (11) is derived from equation (7).

$$\overset{g}{z}_{1i}(t) = \overset{g}{x}_{1i}(t) - x_d = x_{2i}(t) - x_d \quad (11)$$

Based on equation (9) and (10), equation (12) can be obtained.

$$x_{2i}(t) = z_{2i}(t) + \mathbf{V}\mu_{1i}(t) + \mu_{1i}(t) \quad (12)$$

Equation (13), obtained by substituting equation (12) into equation (11).

$$\overset{g}{z}_{1i}(t) = z_{2i}(t) + \mathbf{V}\mu_{1i}(t) + \mu_{1i}(t) - x_d \quad (13)$$

Combining the above formulas, the virtual controller in equation (14) can be obtained.

$$\mu_{1i}(t) = -k_1 z_{1i}(t) - \alpha_1 z_{1i}^p(t) - \beta_1 z_{1i}^q(t) + \lambda_1 \overset{g}{z}_{1i}(t) + \overset{g}{x}_d \quad (14)$$

In equation (14), p, q is a constant. According to the above equation (8) and (14), an auxiliary system of filtering is used in the experiment to compensate its error, which is expressed in equation (15).

$$\begin{cases} \zeta_{1i}(t) = \begin{cases} -\lambda_1 \zeta_{1i}(t) - \alpha_2 \zeta_{1i}^p(t) - \beta_2 \zeta_{1i}^q(t) - \chi_{1i}(t) \zeta_{1i}(t) + \lambda_1 \mathbf{V}\mu_{1i}(t) \\ 0 \end{cases} \\ \|\zeta_{1i}(t)\| > \varpi \\ \|\zeta_{1i}(t)\| < \varpi \end{cases} \quad (15)$$

The aforementioned filters and RBFNN are integrated in the ETM-based formation control method in the experiments. By introducing dynamic variables, the improved flexibility and practical operability of the triggering mechanism can be realized. Mathematical derivation can verify that the formation control mechanism designed in the experiment is stable and capable of autonomously adjusting the required parameters. At the same time, this method is able to obtain a stable fixation time.

The experiment introduces both distributed static ETM and fixed time theory to research multi AUV formation control. A triggering function is constructed, and a formation controller based on distributed static event triggering is designed to achieve global fixed time convergence of the system. The design parameters are the only factors related to this convergence. The algorithm enhances the convergence speed of the formation system while reducing the number of controller triggers and signal transmission frequency. This improves the utilization of limited resources, reduces system energy consumption, and eliminates Zeno behavior. Additionally, the dynamic surface technology introduced in the backstepping method effectively avoids the issue of computational explosion and simplifies the controller design process.

4 Simulation analysis of multi-AUV formation control based on event-triggered and model predictive control

In order to verify the performance of the above-designed MAUVF control method, it is applied in MATLAB R2019a for simulation analysis in the experiment. The experimental environment includes Windows 10 system, Intel® Core TM i5-1 0400F* processor, and running memory of 8GB. The formation control simulation is analyzed by placing multiple AUVs in the obstacle environment in the experiment. The main focus of the study is to verify whether this method can successfully plan smooth obstacle avoidance paths. The path planning results of this formation control method in the presence of obstacles are shown in Figure 5. In the figure, AUV1A denotes the leader and its position coordinates are (0,22), AUV2A~AUV5A are the followers and their coordinates

are (0,19), (0,16), (0,6) and (0,1) respectively. AUV1B~AUV5B denote the reference trajectories. The results in the figure show that AUV2A~AUV5A are able to avoid the obstacles successfully under the lead of the pilot AUV1A. Meanwhile, the motion trajectories of

AUV1A~AUV5A have a high degree of overlap with the reference trajectory. Therefore, under this control method, the multi-formation AUV is able to perform path planning and obstacle avoidance smoothly.

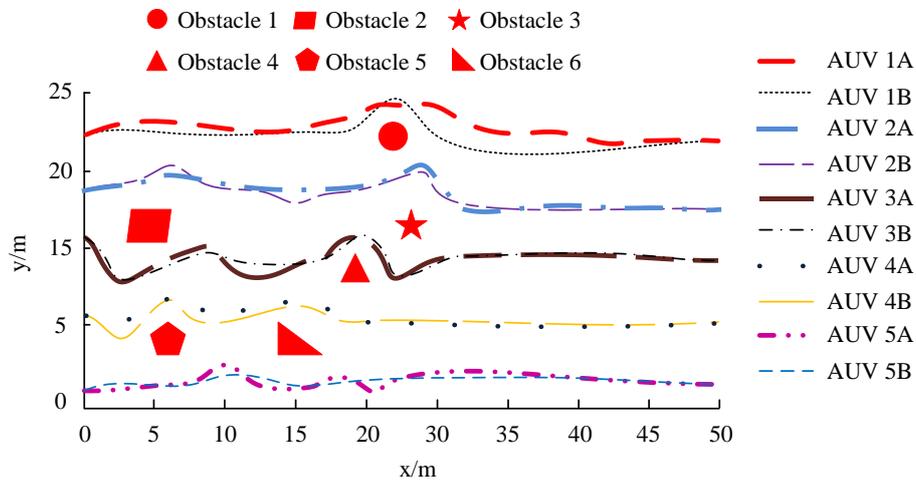


Figure 5: Path planning results for multiple AUV formation

It is put in working condition 1 for simulation and analysis in the study to confirm the efficacy of using the control approach suggested in the experiment. In Figure 6, the motion trajectories of AUV1~AUV5 in three directions in the presence of unknown external perturbations are shown. From the figure, the followers

AUV2~AUV5 are able to keep a certain safe distance to move under the leadership of the navigator AUV1. At the same time, the motion trends of AUV1~AUV5 maintain relative consistency and finally reach the end point smoothly.

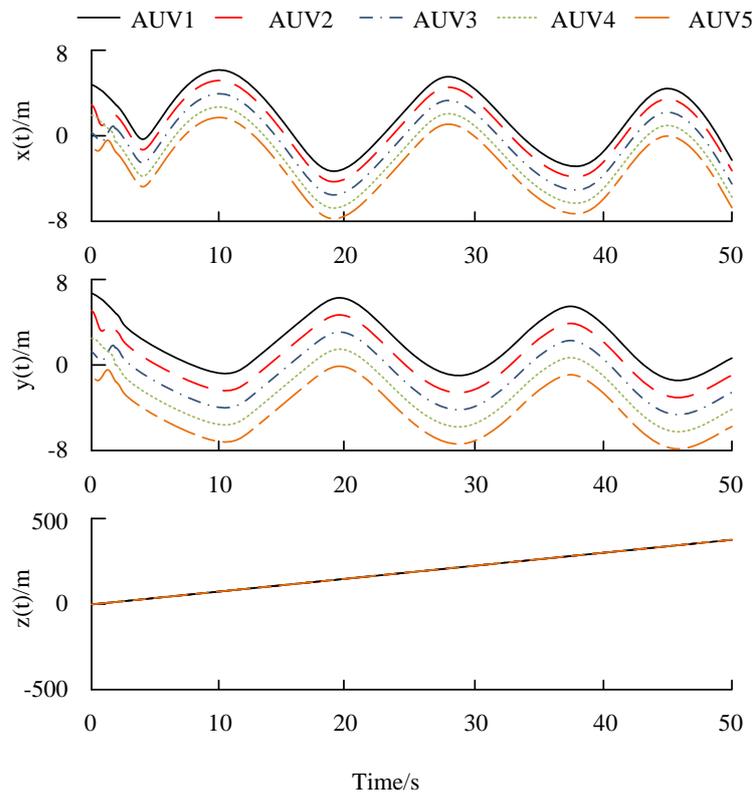


Figure 6: Formation position status of multiple AUV systems

In Figure 7, the formation positions of AUV1~AUV5 in three directions in the working condition 1 environment are shown. During the 50-second moving trajectory, AUV1~AUV5 only produced position errors within the beginning 3 seconds. The maximum error is 3.8 meters.

When the movement trajectory is larger than 3 s, the position error of AUV1~AUV5 is infinitely close to 0. This result indicates that under the leadership of the pilot AUV1, the followers AUV2~AUV5 can reach the designated position smoothly within a safe distance.

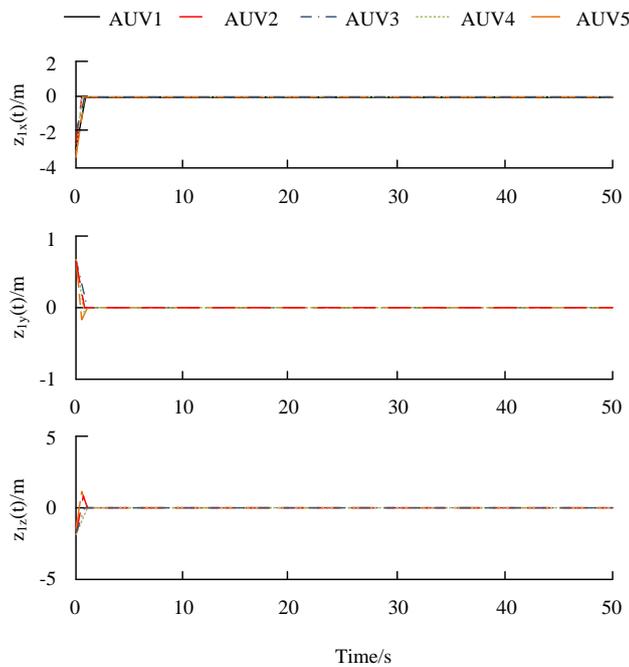


Figure 7: Formation position error of multiple AUV systems

In Figure 8, the velocity information of AUV1~AUV5 during their movement in the working condition 1 environment is demonstrated. From the figure, it can be observed that in the 50-second moving trajectory, the running speeds of AUV1~AUV5 have more obvious fluctuations when the time is lower than 4 seconds. However, when the time is greater than 4 seconds, their

speeds in different directions have some fluctuations, but the whole is in a stable state. Combining the results in Figures 7 and 8, under the leadership of the pilot AUV1, the followers AUV2~AUV5 are able to reach the designated position smoothly and steadily within a safe distance.

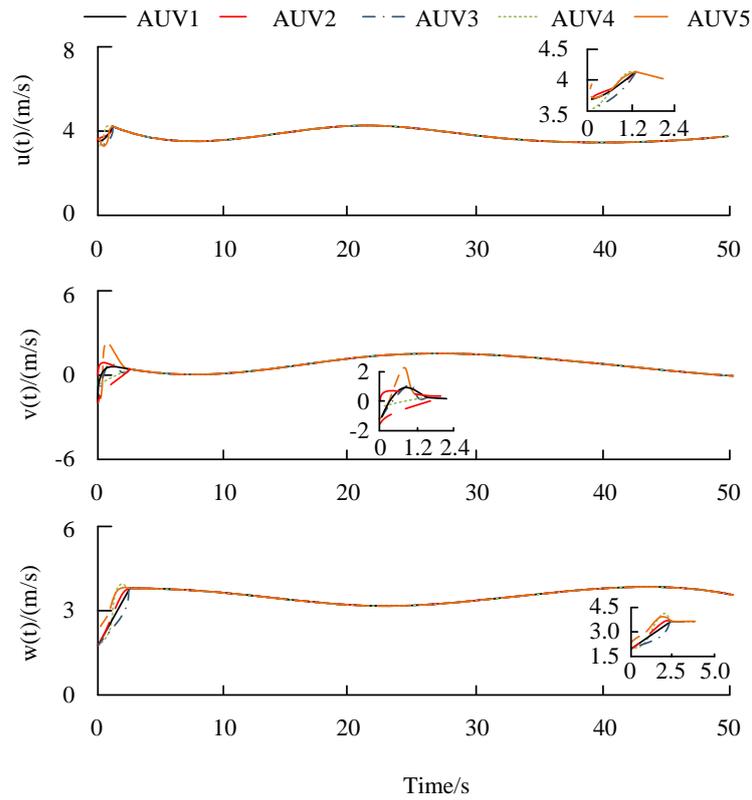


Figure 8: Formation speed status of multiple AUV systems

In Figure 9, the velocity error components of AUV1~AUV5 during the motion in the working condition 1 environment are demonstrated. AUV1~AUV5 have some error fluctuations in the beginning of the motion process. However, their error

ranges are small, and the maximum velocity error is 3.7 m/s. After a period of motion, the velocity errors of AUV1~AUV5 converge to 0. The MAUVF achieves greater formation control during motion, as this result attests.

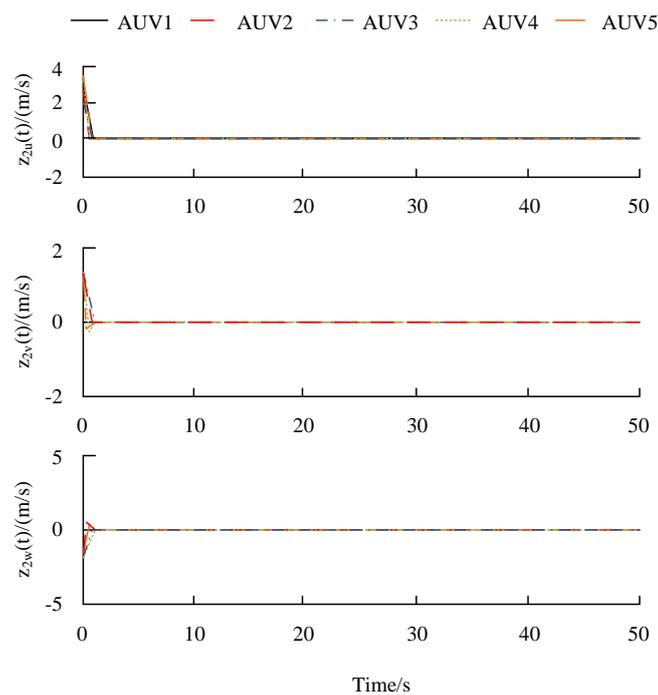


Figure 9: Formation velocity error of multiple AUV systems

The control effect of the MAUVF control method is further tested in the experiment when it is subjected to an unknown external disturbance in Case 1. Figure 10 displays the composite disturbance along with its estimated value for the MAUVF control method in the presence of the disturbance. In this figure, the composite

disturbance of the experimentally proposed formation control method and its estimated value are in overall agreement over the 50-second measurement time. Although there is some prediction error, it is able to satisfy the actual interference detection and realize accurate formation control.

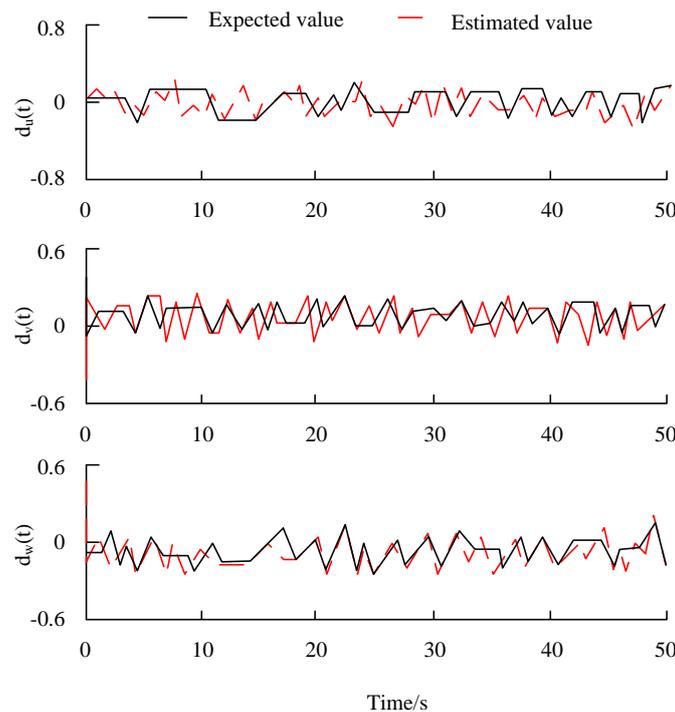


Figure 10: Composite interference and estimated values of multiple AUV systems

Figure 11 shows the timing diagram of AUV1~AUV5 for event triggering in Case 1. From this figure, AUV1~AUV5 have independent triggering times. This can confirm that the method designed in the experiment belongs to the distributed ETM. This triggering method effectively reduces the triggering frequency of the MPC

controller, which in turn reduces the energy consumption of the MAUVF control method. This can improve the utilization time and reduce the cost of AUVs in real applications. This result further confirms the feasibility of the experimentally proposed ETM-based MAUVF control method.

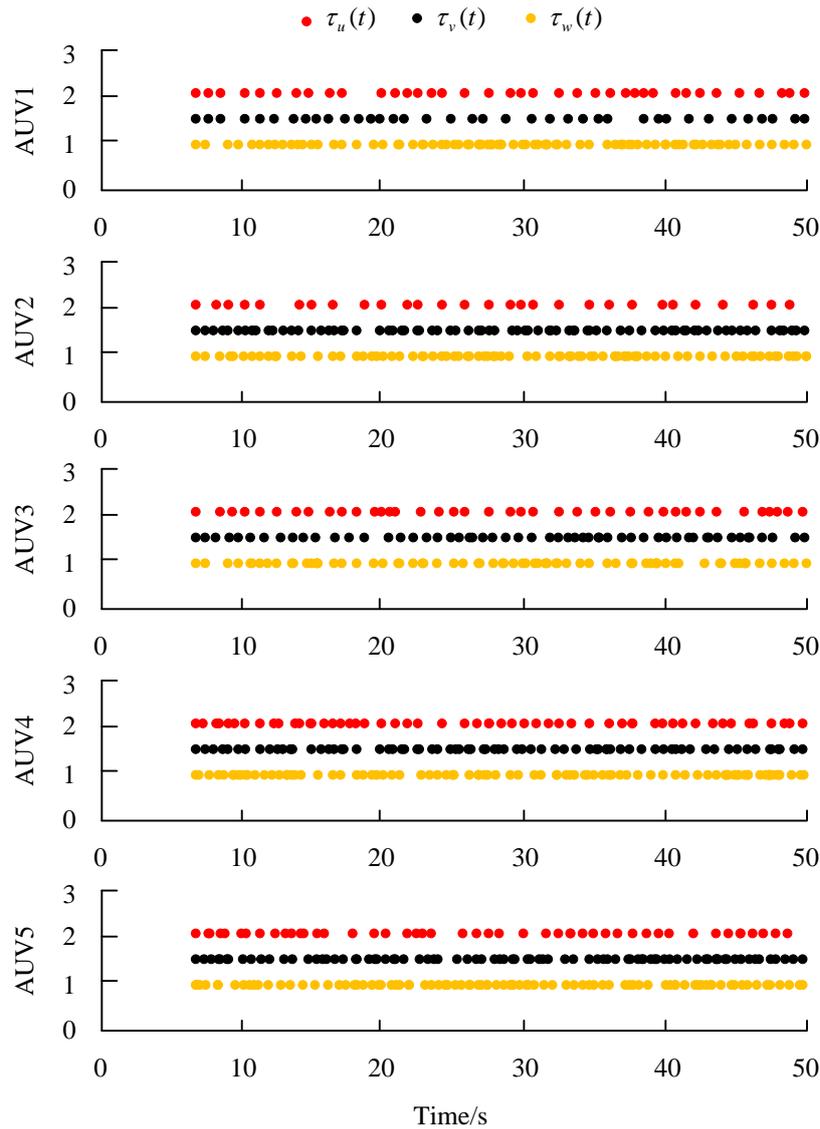


Figure 11: Event triggered timing diagram of multiple AUV systems

The suggested approach is experimentally contrasted with Method 1 based on RBFNN with adaptive control using distributed ETM (Method 2) and backstepping in order to confirm the proposed method's superiority [20-21]. The metrics for the comparison are the Mean Square Error (MSE) of the error signal and its average value. The

suggested technique has the lowest mean and MSE. Its average MSE is 1.4015 and its maximum MSE is 2.4233. This outcome demonstrates how the suggested approach successfully outperforms the comparison method in terms of control performance for the MAUVF control method.

outcomes of several algorithms are displayed in Table 2. The table shows that, out of all the algorithms, the

Table 2: Mean square error and its mean for different methods

MSE	This paper	Method 1	Method 2	Backstepping
z1i_x	1.8244	1.9175	2.6898	2.7447
z1i_y	1.0671	1.1215	2.0024	2.0433
z1i_z	0.5906	0.6208	1.0846	1.1067
z2i_u	0.7306	0.7679	1.7863	1.8228
z2i_v	1.4694	1.5444	1.7791	1.8155

z2i_w	2.4233	2.5469	2.9532	3.0135
Average MSE	This paper	Method 1	Method 2	Backstepping
z1i_x	1.0486	1.0800	1.8656	1.8096
z1i_y	0.6640	0.6839	1.5892	1.5415
z1i_z	0.5957	0.6136	1.0741	1.0418
z2i_u	0.5192	0.5348	1.4504	1.4069
z2i_v	0.8731	0.8993	1.2923	1.2536
z2i_w	1.4015	1.4435	2.3939	2.3221

The number of triggers required by different methods to perform the solution of the optimization problem is compared in Table 3. In the table, the proposed method requires the least number of triggers among all the methods to perform the solution of the optimization problem. For AUV1~AUV5, Pilot AUV1 has the least number of triggers. These results confirm that the proposed method requires less computation than other methods for solving the optimization problem. This effectively saves the computational cost of MAUVF control methods. At the same time, the leader AUV1 is able to derive the optimal solution the fastest, thus effectively leading the other AUVs to meet the realistic operational requirements. Table 3 shows that the DET mechanism reduces the number of controller triggers and update frequencies compared to the distributed static event triggering mechanism by introducing dynamic variables. This is because the

fixed time distributed static event triggering mechanism is designed with a corresponding formation controller, which achieves global fixed time stability of the system. This approach ensures that the convergence time of the formation is not affected by the initial state. Reducing the triggering frequency of the controller and the communication frequency between AUVs can accelerate the convergence speed of the formation. Additionally, unnecessary energy consumption of the system can be effectively reduced, thus avoiding Zeno behavior. The backstepping dynamic surface control algorithm is designed to avoid the problem of 'computational explosion' and simplify the controller design process. This method can effectively reduce system energy consumption and improve the utilization of limited resources in multi-AUV systems.

Table 3: Trigger times of different methods

Working condition 1	This paper	Method 1	Method 2	Backstepping
AUV1	1002	1242	1302	1503
AUV2	1287	1595	1673	1930
AUV3	1379	1709	1792	2068
AUV4	1314	1629	1708	1971
AUV5	1385	1717	1800	2075
Working condition 2	This paper	Method 1	Method 2	Backstepping
AUV1	701	841	833	999
AUV2	900	1070	1081	1297
AUV3	965	1158	1146	1376
AUV4	919	1092	1103	1324
AUV5	969	1163	1151	1382

The experiment's proposed formation method will be used to verify performance in real-world scenarios. The research primarily aimed to determine whether this method can effectively plan an obstacle avoidance path. Figure 12 displays the path planning results of this formation control method in the presence of obstacles. It

is evident from the figure that, under the guidance of navigator AUV1, AUV2~AUV5 successfully navigated through obstacles and reached their destination. The path planning from AUV1 to AUV5 is stable, confirming that the proposed formation method can achieve good results in practical scenarios.

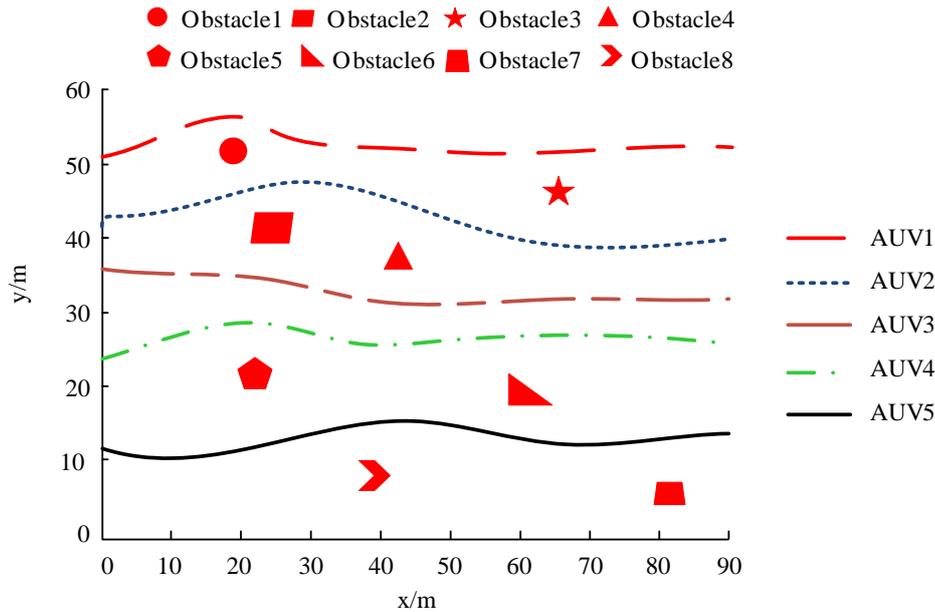


Figure 12: Path planning in real-world scenarios

Taking the above results together, the MAUVF control method based on ETM and MPC proposed in this experiment can realize efficient and accurate control. At the same time, it can reduce the difficulty of computation and resource consumption.

5 Discussion

To enhance the efficiency and stability of multi-AUV formation control, this study proposes combining MPC with ETM. During the experiment, the MPC model was utilized for feedback correction to overcome environmental interference and maintain closed-loop stability. Predictive models are often nonlinear and influenced by unstable factors, hence the addition of a feedback loop during the control process. At the start of each sampling period, the controlled object's output was detected, and the model's prediction results were corrected based on the detection results. Afterward, new optimizations were performed to achieve the desired control effect. Simultaneously, the experimental research of multi-AUV formation control introduced both distributed static ETM and fixed time theory, and constructed a triggering function. In the experiment, a distributed static event-triggered formation controller was designed to achieve global fixed time convergence of the system, which depends solely on the design parameters. The experiment compared the proposed method with adaptive control based on RBFNN, distributed ETM, and backstepping. In the experiment, the proposed method exhibited the lowest MSE and average value compared to all other algorithms. Additionally, it required the fewest number of triggers to solve the optimization problem when compared to the other methods. These results demonstrated that the proposed method is computationally more efficient than the other methods for

solving the optimization problem. This method effectively reduced the computational cost of various AUV formation control methods. Additionally, navigation AUV1 can quickly obtain the optimal solution and effectively guide other AUVs to meet practical work requirements. Compared to the methods in references [5, 7, 12], it was evident that these methods have drawbacks such as high computational complexity, limited application scope, susceptibility to external environmental influences, and poor real-time control performance. This method had a limited range of applications and may be influenced by the research environment, making it ineffective in other scenarios.

6 Conclusion

AUVs are less efficient when operating underwater due to their own resource carrying capacity. MAUVF can make up for its shortcomings when working underwater. However, the existing MAUVF control methods are computationally large, while some methods are only suitable for specific scenarios. To improve the application of MAUVF control methods, ETM and MPC are introduced in the experiment for the design of formation control methods. To reduce the difficulty of computation, the methods were optimized in the experiments using methods such as RBFNN and filter control. The above experimental results confirmed that AUV2A~AUV5A were able to avoid obstacles smoothly under the lead of the pilot AUV1A. The motion trajectories of AUV1A~AUV5A had a high degree of overlap with the reference trajectory. AUV1~AUV5 produced a position error within 3 seconds of motion, and it had a maximum error of 3.8 meters. When the motion trajectory was greater than 3 seconds, the position errors of AUV1~AUV5 tended to be infinitely close to 0.

AUV1~AUV5 showed some error fluctuations at the beginning of the motion. But their error ranges were small, and the maximum velocity error was 3.7 m/s. After a period of motion, the velocity errors of AUV1~AUV5 were converged to 0. The proposed method had the lowest MSE and its average value among all the algorithms. Its maximum MSE was 2.4233 and its average MSE was 1.4015. The proposed method required the least number of triggers for AUV1~AUV5 to perform the solution of the optimization problem among all the methods. For AUV1~AUV5, Pilot AUV1 had the least number of triggers. The aforementioned results confirm that the proposed MAUVF control method based on ETM and MPC is able to realize efficient and accurate control and reduce the difficulty of computation and resource consumption. Although the experiments show that the MAUVF control method has been effectively improved, there are still some problems. For example, the experiments have not yet been analyzed by simulation in complex environments. This may limit the application of MAUVF. It is necessary to further deepen the experiments to improve the application of MAUVF in future research. Due to limitations in experimental conditions, it is currently not possible to simulate experiments in real environments and complex scenes, which can serve as a direction for future research.

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References

- [1] C. Meng, W. Zhang, and X. Du, "Finite-time extended state observer based collision-free leaderless formation control of multiple AUVs via event-triggered control," *Ocean Engineering*, vol. 268, no. Jan.15, pp. 113605, 2023. <https://doi.org/10.1016/j.oceaneng.2022.113605>
- [2] M. Narang, M.C. Joshi, A.K. Pal, "A hesitant fuzzy multiplicative Base-criterion multi-criteria group decision making method," *Informatica*, vol. 46, pp. 235-242, 2022. <https://doi.org/10.31449/inf.v46i2.3452>
- [3] B. K. Singh, and A. Kumar, "Model predictive control using LPV approach for trajectory tracking of quadrotor UAV with external disturbances," *Aircraft engineering and aerospace technology*, vol. 95, no. 4, pp. 607-618, 2023. <https://doi.org/10.1108/AEAT-12-2021-0368>
- [4] T. Liu, C. Wang, J. Zhang, and J. Qiao, "Fully distributed event-triggered asymptotic attitude coordination of multiple spacecraft systems with disturbances," *International Journal of Robust and Nonlinear Control*, vol. 33, no. 1, pp. 466-488, 2023. <https://doi.org/10.1002/rnc.6437>
- [5] M. Liu, X. Zhuo, Y. Yuan, Y. Lu, Y. Wei, X. Tu, and F. Qu, "Adaptive scheduling MAC protocol in underwater acoustic broadcast communications for AUV formation," *IEEE Internet of Things Journal*, vol. 10, no. 8, pp. 6887-6901, 2023. <https://doi.org/10.1109/JIOT.2022.3227265>
- [6] W. Cao, J. Yan, X. Yang, X. Luo, and X. Guan, "Communication-aware formation control of AUVs with model uncertainty and fading channel via integral reinforcement learning," *IEEE/CAA Journal of Automatica Sinica*, vol. 10, no. 1, pp. 159-176, 2023. <https://doi.org/10.1109/JAS.2023.123021>
- [7] W. Pang, D. Zhu, and S. X. Yang, "A novel time-varying formation obstacle avoidance algorithm for multiple AUVs," *International Journal of Robotics and Automation*, vol. 38, no. 3, pp. 194-207, 2023. <https://doi.org/10.2316/j.2023.206-0845>
- [8] J. C. Yu, K. Chen, and J. Zhang, "Intelligent control method for AUV formation under different communication and positioning methods: A review," *Journal of Unmanned Undersea Systems*, vol. 31, no. 1, pp. 30-37, 2023. <https://doi.org/10.11993/j.issn.2096-3920.2022-0079>
- [9] G. Angelov, A. D. Corella, V. M. Veliov, "On the accuracy of the model predictive control method," *SIAM Journal on Control and Optimization*, vol. 60, no. 4, pp. 2469-2487, 2022. <https://doi.org/10.1137/21M1460430>
- [10] H. Zhou, P. Feng, and W. Chou, "A hybrid obstacle avoidance method for mobile robot navigation in unstructured environment," *Industrial Robot*, vol. 51, no. 1, pp. 94-106, 2023. <https://doi.org/10.1108/ir-04-2022-0102>
- [11] Z. Liu, D. Zhu, C. Liu, and S. X. Yang, "A novel path planning algorithm of AUV with model predictive control," *International Journal of Robotics and Automation*, vol. 37, no. 6, pp. 460-467, 2022. <https://doi.org/10.2316/J.2022.206-0710>
- [12] P. Gong, Z. Yan, W. Zhang, and J. Tang, "Trajectory tracking control for autonomous underwater vehicles based on dual closed-loop of MPC with uncertain dynamics," *Ocean Engineering*, vol. 265, no. Dec.1, pp. 1532-1541, 2022. <https://doi.org/10.1016/j.oceaneng.2022.112697>
- [13] Y. Bian, J. Zhang, M. Hu, C. Du, Q. Cui, and R. Ding, "Self-triggered distributed model predictive control for cooperative diving of multi-AUV system," *Ocean Engineering*, vol. 267, no. Jan.1,

- pp. 1-14, 2023.
<https://doi.org/10.1016/j.oceaneng.2022.113262>
- [14] F. Zhang, Y. Y. Chen, and Y. Zhang, "Finite-time event-triggered containment control of multiple Euler–Lagrange systems with unknown control coefficients," *Journal of the Franklin Institute*, vol. 360, no. 2, pp. 777-791, 2023.
<https://doi.org/10.1016/j.jfranklin.2022.11.043>
- [15] W. Li, "Formation control of a multi-autonomous underwater vehicle event-triggered mechanism based on the Hungarian algorithm," *Machines*, vol. 9, no. 12, pp. 346-372, 2021.
<https://doi.org/10.3390/machines9120346>
- [16] Z. Su, X. Lin, B. Huang, D. Zhao, and H. Sun, "Improved dynamic event-triggered anti-unwinding control for autonomous underwater vehicles," *Ocean Engineering*, vol. 272, no. Mar. 15, pp. 1-19, 2023.
<https://doi.org/10.2139/ssrn.4278486>
- [17] L. Wang, D. Zhu, W. Pang, and C. Luo, "A novel obstacle avoidance consensus control for Multi-AUV formation system," *IEEE/CAA Journal of Automatica Sinica*, vol. 15, no. 5, pp. 1304-1318, 2023.
<https://doi.org/10.1109/JAS.2023.123201>
- [18] R. R. Hossain, and R. Kumar, "Machine learning accelerated real-time model predictive control for power systems," *IEEE/CAA Journal of Automatica Sinica*, vol. 10, no. 4, pp. 916-930, 2023.
<https://doi.org/10.1109/JAS.2023.123135>
- [19] A. D. Carnerero, D. R. Ramirez, D. Limon, and T. Alamo, "Kernel-based state-space kriging for predictive control," *IEEE/CAA Journal of Automatica Sinica*, vol. 10, no. 5, pp. 1263-1275, 2023.
<https://doi.org/10.1109/JAS.2023.123459>
- [20] J. Xu, Y. Cui, Z. Yan, F. Huang, X. Du, and D. Wu, "Event-triggered adaptive target tracking control for an underactuated autonomous underwater vehicle with actuator faults," *Journal of the Franklin Institute*, vol. 360, no. 4, pp. 2867-2892, 2023.
<https://doi.org/10.1016/j.jfranklin.2023.01.020>
- [21] I. Banno, S. I. Azuma, R. Ariizumi, and T. Asai, "Sparse event-triggered control of linear systems," *International Journal of Robust and Nonlinear Control*, vol. 33, no. 1, pp. 134-158, 2023.
<https://doi.org/10.1002/rnc.6278>

