A Mixed Noise Removal Method Based on Total Variation

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Due to the technology limits, digital images always include some defects, such as noise. Noise reduces image quality and affects the result of image processing. While in most cases, noise has Gaussian distribution, in biomedical images, noise is usually a combination of Poisson and Gaussian noises. This combination is naturally considered as a superposition of Gaussian noise over Poisson noise. In this paper, we propose a method to remove such a type of mixed noise based on a novel approach: we consider the superposition of noises like a linear combination. We use the idea of the total variation of an image intensity (brightness) function to remove this combination of noises.

Povzetek: Članek predlaga izvirno kombinacijo Gaussovega in Poissonovega filtra za filtriranje šuma v slikah.

1 Introduction

Image denoising has attracted a lot of attention in recent years. In order to suppress noise effectively, we need to know its type. There are many types of noises, for example, Gaussian (digital images), Poisson (X-Ray images), Speckle (ultra sonograms) noises and so on.

One of the most famous effective methods is the total variation model [2-4, 10, 12, 17, 18, 22, 26]. The first person who suggested it to solve the denoising problem is Rudin [17]. He used the total variation as a universal tool in image processing. His denoising model is well-known as the ROF model [3, 17]. The ROF model is targeted to efficiently remove Gaussian noise only.

This model is often used to remove not only Gaussian noise, but also other types of noise. For example, the ROF model suppresses Poisson noise not so effectively. Le T. [9] proposed another model, well-known as the modified ROF model to remove Poisson noise only.

Gaussian and Poisson noises both are widespread in real situations, but their combination is important too, for example, in electronic microscopy images [7, 8]. In these images, both types of noises are combined as a superposition. In physical process, Poisson noise usually is added first, before Gaussian noise. Luisier F. with co-authors proposed the theoretically strong PURE-LET method [11] (Poisson-Gaussian unbiased risk estimate – linear expansion of thresholds) to remove this type of combination of noises.

However, such kind of noises usually can be considered as dependent on the image acquisition systems. At the same time, in many papers devoted to the image denoising problem the idea of Poisson-Gaussian noise combination is considered, even though such is not the case.

From other side, many noise reduction approaches have been developed, particularly, wavelet-based transforms, etc. It needs to draw attention, noise reduction approaches that have been developed based on wavelet transform are only for Gaussian or Poisson noise.

In order to remove mixed noise, let us assume that the superposition of noises can be equivalent to some unknown linear combination of them.

We can combine ROF and modified ROF models to suppress the linear combination of noises. The obtained model is supposed to remove the mixed noise better than ROF or modified ROF models separately. Additionally, it is simpler than PURE-LET, because we try to find only the proportion between Poisson and Gaussian noises in the mixed noise.

In experiments, we use images and add noise into them. The image quality is compared with some other denoising methods such as ROF, modified ROF models, and PURE-LET method to remove the superposition of noises. In our paper [19], we proposed to remove the linear combination of Poisson and Gaussian noises and compared results with Wiener [1] and median [23] filters, and with Beltrami method [29]. Our method gives better results for Gaussian and Poisson noises separately, and for the combination of noises too. Hence, in this paper, we do not compare our approach with these methods.

In order to compare image quality after restoration, we use criteria PSNR (Peak Signal-to-Noise Ratio), MSE (Mean Square Error), SSIM (Structure SIMilarity) [24, 25]. The PSNR criterion is the most important, because it is always used to evaluate images and signals quality.

In this paper, we try to represent and discuss only the case limited by the greyscale artificial and real images with artificial noise. According to it, we can use only criteria above based on the full-reference image quality evaluation approach.

In the case of greyscale real images with unknown noises, we need to use the no-reference approach to evaluate the quality of denoising. In general, it is complicated theoretical problem to develop a criterion for it.

Our investigation based on BRISQUE criterion [13] (Blind/Referenceless Image Spatial QUality Evaluator) in this case was discussed in paper [20].

2 Combined denoising model

Let in R^2 space a bounded domain $\Omega \subset \mathrm{R}^2$ be given. Let functions $u(x,y) \in \mathrm{R}$ and $v(x,y) \in \mathrm{R}$, respectively, be ideal (without noise) and observed (noisy) images, $(x,y) \in \Omega$. For smooth function u, its total variation can be defined by $V_T[u] = \int_\Omega |\nabla u|\, dxdy$, where $\nabla u = (u_x, u_y)$ is a gradient, $u_x = \partial u / \partial x$, $u_y = \partial u / \partial y$, $|\nabla u| = \sqrt{u_x^2 + u_y^2}$. In this paper, we consider that the function u always has limited total variation $V_T[u] < \infty$.

According to [2, 3, 17, 18], an image smoothness is characterized by the total variation of an image intensity function. The total variation of the noisy image is always greater than the total variation of the corresponding smooth image. In order to solve the problem $V_T[u] \to \min$, we need to use the following condition

$$\int_\Omega (v-u)^2\, dxdy = const\,.$$

Hence, we obtain the ROF model to remove Gaussian noise in the image [17, 18]:

$$u^* = \arg\min_u \left(\int_\Omega |\nabla u|\, dxdy + \frac{\lambda}{2} \int_\Omega (v-u)^2\, dxdy \right),$$

where $\lambda > 0$ is Lagrange multiplier. This is a solution of the unconstrained optimization problem.

In order to remove Poisson noise, Le T. built another model based on ROF model [9] as the optimization problem $V_T[u] \to \min$ with the following constraint

$$\int_\Omega \ln(p(v\,|\,u))\,dxdy = \int_\Omega (u - v\ln(u))\,dxdy = const\,.$$

This model resulted in the following unconstrained optimization problem

$$u^* = \arg\min_u \left(\int_\Omega |\nabla u|\, dxdy + \beta \int_\Omega (u - v\ln(u))\,dxdy \right),$$

where $\beta > 0$ is a coefficient of regularization. This is the known modified ROF model to remove Poisson noise.

In order to build a model for removing the mixed Poisson-Gaussian noise, we also solve the same optimization problem $V_T[u] \to \min$, but with a different constraint as follows.

This constraint is very similar to constraints above. We consider, the noise variance is unchangeable (Poisson noise is not changed and Gaussian noise only depends on noise variance):

$$\int_\Omega \ln(p(v\,|\,u))\,dxdy = const\,, \tag{1}$$

where $p(v\,|\,u)$ is a conditional probability of the real image v with the ideal image u given.

The probability density function of Gaussian noise is

$$p_1(v\,|\,u) = \exp\left(-\frac{(v-u)^2}{2\sigma^2} \right) / (\sigma\sqrt{2\pi})\,,$$

and the probability distribution of Poisson noise is

$$p_2(v\,|\,u) = \exp(-u)u^v / v!\,.$$

We have to notice that intensity functions of images u and v are integer (for example, for 8-bits greyscale image the range of intensity is from 0 to 255).

In order to combine Gaussian and Poisson noises, we consider the following linear combination

$$\ln(p(v\,|\,u)) = \lambda_1 \ln(p_1(v\,|\,u)) + \lambda_2 \ln(p_2(v\,|\,u))\,,$$

$$\lambda_1 > 0,\ \lambda_2 > 0,\ \lambda_1 + \lambda_2 = 1\,.$$

According to (1), we obtain the denoising problem as a constrained optimization problem

$$\begin{cases} u^* = \arg\min_u \int_\Omega |\nabla u|\, dxdy \\ \int_\Omega \left(\frac{\lambda_1}{2\sigma^2}(v-u)^2 + \lambda_2 (u - v\ln(u)) \right) dxdy = \kappa\,, \end{cases}$$

where κ is a constant value. We transform this problem into unconstrained optimization problem by using Lagrange functional

$$L(u,\tau) = \int_\Omega |\nabla u|\, dxdy + \tau \left(\frac{\lambda_1}{2\sigma^2} \int_\Omega (v-u)^2\, dxdy + \lambda_2 \int_\Omega (u - v\ln(u))\,dxdy - \kappa \right)$$

to find the solution as

$$(u^*, \tau^*) = \arg\min_{u,\tau} L(u,\tau) \tag{2}$$

where $\tau > 0$ is Lagrange multiplier.

If $\lambda_1 = 0$ and $\beta = \lambda_2 \tau$, we obtain the modified ROF model to remove Poisson noise. If $\lambda_2 = 0$ and $\lambda = \lambda_1 \tau / \sigma^2$, we obtain the ROF model to remove Gaussian noise. If $\lambda_1 > 0, \lambda_2 > 0$, we obtain our model to remove mixed Poisson-Gaussian noise.

3 Discrete denoising model

The problem (2) can be solved by using Lagrange multipliers method [5, 16, 28].

We use Euler-Lagrange equation [28]. Let a function $f(x,y)$ be defined in a limited domain $\Omega \subset R^2$ and be second-order continuously differentiated by x and y, where $(x,y) \in \Omega$. Let $F(x,y,f,f_x,f_y)$ be a convex functional, where $f_x = \partial f / \partial x$, $f_y = \partial f / \partial y$. Then the solution of the following optimization problem

$$\int_\Omega F(x,y,f,f_x,f_y)dxdy \to \min$$

satisfies the following Euler-Lagrange equation

$$F_f(x,y,f,f_x,f_y) - \frac{\partial}{\partial x}F_{f_x}(x,y,f,f_x,f_y) - \frac{\partial}{\partial y}F_{f_y}(x,y,f,f_x,f_y) = 0,$$

where $F_f = \partial F / \partial f$, $F_{f_x} = \partial F / \partial f_x$, $F_{f_y} = \partial F / \partial f_y$.

We use the above result to solve the obtained model. Then the solution of the problem (2) satisfies the following Euler-Lagrange equation

$$-\frac{\lambda_1}{\sigma^2}(v-u) + \lambda_2(1-\frac{v}{u}) - \mu\frac{\partial}{\partial x}\left(\frac{u_x}{\sqrt{u_x^2+u_y^2}}\right) - \mu\frac{\partial}{\partial y}\left(\frac{u_y}{\sqrt{u_x^2+u_y^2}}\right) = 0, \tag{3}$$

where $\mu = 1/\tau$. We rewrite (3) in the form

$$\frac{\lambda_1}{\sigma^2}(v-u) - \lambda_2(1-\frac{v}{u}) + \mu\frac{u_{xx}u_y^2 - 2u_xu_yu_{xy} + u_x^2u_{yy}}{(u_x^2+u_y^2)^{3/2}} = 0 \tag{4}$$

$$u_{xx} = \frac{\partial^2 u}{\partial x^2},\quad u_{yy} = \frac{\partial^2 u}{\partial y^2},\quad u_{xy} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right) = u_{yx}.$$

In order to obtain the discrete form of the model (4), we add an artificial time parameter and consider the function $u = u(x,y,t)$ in the following diffusion equation

$$u_t = \frac{\partial u}{\partial t} = \frac{\lambda_1}{\sigma^2}(v-u) - \lambda_2(1-\frac{v}{u}) + \mu\frac{u_{xx}u_y^2 - 2u_xu_yu_{xy} + u_x^2u_{yy}}{(u_x^2+u_y^2)^{3/2}}. \tag{5}$$

Then the discrete form of the equation (5) is

$$u_{ij}^{k+1} = u_{ij}^k + \xi\left(\frac{\lambda_1}{\sigma^2}(v_{ij} - u_{ij}^k) - \lambda_2(1-\frac{v_{ij}}{u_{ij}^k}) + \mu\varphi_{ij}^k\right), \tag{6}$$

$$\varphi_{ij}^k = \frac{\nabla_{xx}(u_{ij}^k)(\nabla_y(u_{ij}^k))^2}{((\nabla_x(u_{ij}^k))^2 + (\nabla_y(u_{ij}^k))^2)^{3/2}} + \frac{-2\nabla_x(u_{ij}^k)\nabla_y(u_{ij}^k)\nabla_{xy}(u_{ij}^k) + (\nabla_x(u_{ij}^k))^2\nabla_{yy}(u_{ij}^k)}{((\nabla_x(u_{ij}^k))^2 + (\nabla_y(u_{ij}^k))^2)^{3/2}},$$

$$\nabla_x(u_{ij}^k) = \frac{u_{i+1,j}^k - u_{i-1,j}^k}{2\Delta x},$$

$$\nabla_y(u_{ij}^k) = \frac{u_{i,j+1}^k - u_{i,j-1}^k}{2\Delta y},\ \nabla_{xx}(u_{ij}^k) = \frac{u_{i+1,j}^k - 2u_{ij}^k + u_{i-1,j}^k}{(\Delta x)^2},$$

$$\nabla_{yy}(u_{ij}^k) = \frac{u_{i,j+1}^k - 2u_{ij}^k + u_{i,j-1}^k}{(\Delta y)^2},$$

$$\nabla_{xy}(u_{ij}^k) = \frac{u_{i+1,j+1}^k - u_{i+1,j-1}^k - u_{i-1,j+1}^k + u_{i-1,j-1}^k}{4\Delta x\Delta y},$$

$$u_{0j}^k = u_{1j}^k;\ u_{N_1+1,j}^k = u_{N_1,j}^k;\ u_{i0}^k = u_{i1}^k;\ u_{i,N_2+1}^k = u_{i,N_2}^k;$$

$$i = 1,...,N_1;\ j = 1,...,N_2;$$

$$k = 0,1,...,K;\ \Delta x = \Delta y = 1;\ 0 < \xi < 1,$$

where K is enough great number, $K = 500$.

4 Optimal model parameters

In practice, parameters $\lambda_1, \lambda_2, \mu, \sigma$ in procedure (6) are usually unknown. We have to change $\lambda_1, \lambda_2, \mu$ into $\lambda_1^k, \lambda_2^k, \mu^k$ to evaluate them on every iteration step k.

4.1 Optimal parameters λ1 and λ2

Let (u,τ) be a solution of problem (2). Then we obtain the following condition $\partial L(u,\tau)/\partial u = 0$. This condition give us optimal λ_1 and λ_2:

$$\lambda_1 = \frac{\int_\Omega (1-\frac{v}{u})dxdy}{\frac{1}{\sigma^2}\int_\Omega (v-u)dxdy + \int_\Omega (1-\frac{v}{u})dxdy},\ \lambda_2 = 1 - \lambda_1.$$

The discrete form for $k = 0,1,...,K$ is

$$\lambda_1^k = \frac{\sum_{i=1}^{N_1}\sum_{j=1}^{N_2}(1-\frac{v_{ij}}{u_{ij}^k})}{\sum_{i=1}^{N_1}\sum_{j=1}^{N_2}(\frac{v_{ij}-u_{ij}^k}{\sigma^2} + 1 - \frac{v_{ij}}{u_{ij}^k})},\ \lambda_2^k = 1 - \lambda_1^k.$$

4.2 Optimal parameter μ

In order to find the optimal μ, we multiply (3) by $(v-u)$ and integrate by parts over domain Ω. Finally, we obtain the formula to find the optimal

$$\mu = \frac{\int_\Omega (-\frac{\lambda_1}{\sigma^2}(v-u)^2 - \lambda_2\frac{(v-u)^2}{u})dxdy}{\int_\Omega (\sqrt{u_x^2+u_y^2} - \frac{u_xv_x + u_yv_y}{\sqrt{u_x^2+u_y^2}})dxdy}.$$

The discrete form is

$$\mu^k = \frac{\sum_{i=1}^{N_1}\sum_{j=1}^{N_2}(-\frac{\lambda_1^k}{\sigma^2}(v_{ij}-u_{ij}^k)^2 - \lambda_2^k\frac{(v_{ij}-u_{ij}^k)^2}{u_{ij}^k})}{\sum_{i=1}^{N_1}\sum_{j=1}^{N_2}\eta_{ij}^k},$$

$$\eta_{ij}^k = \sqrt{(\nabla_x(u_{ij}^k))^2 + (\nabla_y(u_{ij}^k))^2} - \frac{\nabla_x(u_{ij}^k)\nabla_x(v_{ij}) + \nabla_y(u_{ij}^k)\nabla_y(v_{ij})}{\sqrt{(\nabla_x(u_{ij}^k))^2 + (\nabla_y(u_{ij}^k))^2}},$$

$$\nabla_x(u_{ij}^k) = \frac{u_{i+1,j}^k - u_{i-1,j}^k}{2\Delta x},\ \nabla_y(u_{ij}^k) = \frac{u_{i,j+1}^k - u_{i,j-1}^k}{2\Delta y},$$

$$\nabla_x(v_{ij}^k) = \frac{v_{i+1,j}^k - v_{i-1,j}^k}{2\Delta x} v,\ \nabla_y(v_{ij}^k) = \frac{v_{i,j+1}^k - v_{i,j-1}^k}{2\Delta y},$$

$$u_{0j}^k = u_{1j}^k;\ u_{N_1+1,j}^k = u_{N_1,j}^k;\ u_{i0}^k = u_{i1}^k;\ u_{i,N_2+1}^k = u_{i,N_2}^k;$$

$$v_{0j} = v_{1j};\ v_{N_1+1,j} = v_{N_1 j};\ v_{i0} = v_{i1};\ v_{i,N_2+1} = v_{i,N_2};$$

$$i = 1,\dots,N_1;\ j = 1,\dots,N_2;\ k = 0,1,\dots,K;\ \Delta x = \Delta y = 1.$$

4.3 Optimal parameter σ

The parameter σ is calculated at the first step of the iteration process. We use the method of Immerker [6]:

$$\sigma = \frac{\sqrt{\pi/2}}{6(N_1-2)(N_2-2)} \sum_{i=1}^{N_1}\sum_{j=1}^{N_2} |u_{ij} * \Lambda|,$$

with the mask $\Lambda = \begin{pmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{pmatrix}$ for convolution operator *,

$$u_{ij} * \Lambda = u_{i-1,j-1}\Lambda_{33} + u_{i,j-1}\Lambda_{32} + u_{i+1,j-1}\Lambda_{31} + u_{i-1,j}\Lambda_{23} + u_{ij}\Lambda_{22} + u_{i+1,j}\Lambda_{21} + u_{i-1,j+1}\Lambda_{13} + u_{i,j+1}\Lambda_{12} + u_{i+1,j+1}\Lambda_{11},$$

$$i = 1,\dots,N_1;\ j = 1,\dots,N_2;$$

$u_{ij} = 0$, if $i = 0$, or $j = 0$, or $i = N_1 + 1$, or $j = N_2 + 1$.

4.4 Initial solution

In the iteration procedure (6), the result depends on initial parameters $\lambda_1^0, \lambda_2^0, \mu^0$. If $\lambda_1^0, \lambda_2^0, \mu^0$ are given first, then its unsuitable values define not so good solution u_{ij} and later, not so good evaluation of a probability distribution parameters. If $\lambda_1^0, \lambda_2^0, \mu^0$ are randomized, the result is unacceptable too, because of the additional noise added in the image.

Of course, initial values of $\lambda_1^0, \lambda_2^0, \mu^0$ need to be closed to required values. We evaluate $\lambda_1^0, \lambda_2^0, \mu^0$ as average values of neighbour pixels of the image, for example, by the method of Immerker.

5 Image quality evaluation

In order to evaluate the image quality after denoising, we use criteria PSNR, MSE and SSIM [24, 25]:

$$Q_{\mathrm{MSE}} = \frac{1}{N_1 N_2}\sum_{i=1}^{N_1}\sum_{j=1}^{N_2}(v_{ij} - u_{ij})^2,\ Q_{\mathrm{PSNR}} = 10\lg\left(\frac{L^2}{Q_{\mathrm{MSE}}}\right),$$

$$Q_{\mathrm{SSIM}} = \frac{(2\bar{u}\,\bar{v} + C_1)(2\sigma_{uv} + C_2)}{(\bar{u}^2 + \bar{v}^2 + C_1)(\sigma_u^2 + \sigma_v^2 + C_2)},$$

where

$$\bar{u} = \frac{1}{N_1 N_2}\sum_{i=1}^{N_1}\sum_{j=1}^{N_2} u_{ij},\ \bar{v} = \frac{1}{N_1 N_2}\sum_{i=1}^{N_1}\sum_{j=1}^{N_2} v_{ij}.$$

$$\sigma_u^2 = \frac{1}{N_1 N_2 - 1}\sum_{i=1}^{N_1}\sum_{j=1}^{N_2}(u_{ij} - \bar{u})^2,$$

$$\sigma_v^2 = \frac{1}{N_1 N_2 - 1}\sum_{i=1}^{N_1}\sum_{j=1}^{N_2}(v_{ij} - \bar{v})^2,$$

$$\sigma_{uv} = \frac{1}{N_1 N_2 - 1}\sum_{i=1}^{N_1}\sum_{j=1}^{N_2}(u_{ij} - \bar{u})(v_{ij} - \bar{v}),$$

$$C_1 = (K_1 L)^2, C_2 = (K_2 L)^2;\ K_1 \ll 1;\ K_2 \ll 1.$$

For example, $K_1 = K_2 = 10^{-6}$, L is an image intensity with $L = 255$ for 8-bits greyscale image.

The greater the value of Q_{PSNR}, the better the image quality. If the value of Q_{PSNR} belongs to the interval from 20 to 25, then the image quality is acceptable, for example, for wireless transmission [21].

The Q_{MSE} is a mean squared error and is used to evaluate the difference between two images. The lower the value of Q_{MSE}, the better the result of restoration. The value of Q_{MSE} directly related to the value of Q_{PSNR}.

The value of Q_{SSIM} is used to evaluate an image quality by comparing the similarity of two images. This value is between -1 and 1. The greater the value of Q_{SSIM}, the better the image quality.

6 Experiments and discussion

In this paper, we consider cases as in [19] and additionally the superposition of noises. The image size is changed from 300x300 pixels to 256x256 pixels specified in PURE-LET method [11]. We process the artificial image with artificial noise and the real image with artificial noise. The artificial image is noise-free and we need to add noise with high intensity (the image to be very noisy) to reduce its quality. Therefore, we specify 0.6 for proportion of Gaussian noisy image and 0.4 for proportion of Poisson noisy image. The real image (captured by a digital device) already includes some noise. We specify 0.5 for proportion of Gaussian noisy image and 0.5 for proportion of Poisson noisy image too.

We need to point the attention in the case of Gaussian noise our method sometimes can be better than ROF, because the method to evaluate the variance of Gaussian noise can be better than one included in the original ROF model in many cases. In the case of superposition of noises, our method sometimes can be better than PURE-LET, because parameters of our method are usually more optimal than in original model too.

6.1 Artificial image with artificial noise

We use artificial image with artificial mixed noise for the first test. The image includes eight bars (Fig. 1a). Other images (Fig. 1b-j) show noisy and denoised images and zoomed out part of them.

Artificial noise is generated by linear combination, and by superposition of Poisson and Gaussian noises.

For both cases, we consider Poisson noise first. Its probability density is $p_2(v\,|\,u)$, and variance is $\sigma_2 = \sqrt{u_{ij}}$

at every pixel (i, j), $i = 1,...,N_1$, $j = 1,...,N_2$. Poisson noise variance is an average value $\bar{\sigma}_2 = 11.7939$. If the grey value of a pixel after adding of Poisson noise is out of the interval from 0 to 255, it needs to be reset to $v^{(2)}_{ij} = u_{ij}$. For this image, there are no pixels out of this interval. Next, we consider the variance of Gaussian noise is four times greater than the variance of Poisson noise $\sigma_1 = 4\bar{\sigma}_2 = 47.1757$.

For the linear combination of noises, we denote the intensity function of Gaussian noisy image as $v^{(1)}$. As above, values of $v^{(1)}$ need to be between 0 and 255. If the grey value of a pixel after adding of Gaussian noise is out of the interval from 0 to 255, it needs to be reset to $v^{(1)}{}_{ij} = u_{ij}$. In this case, there are 1075 pixels out of this

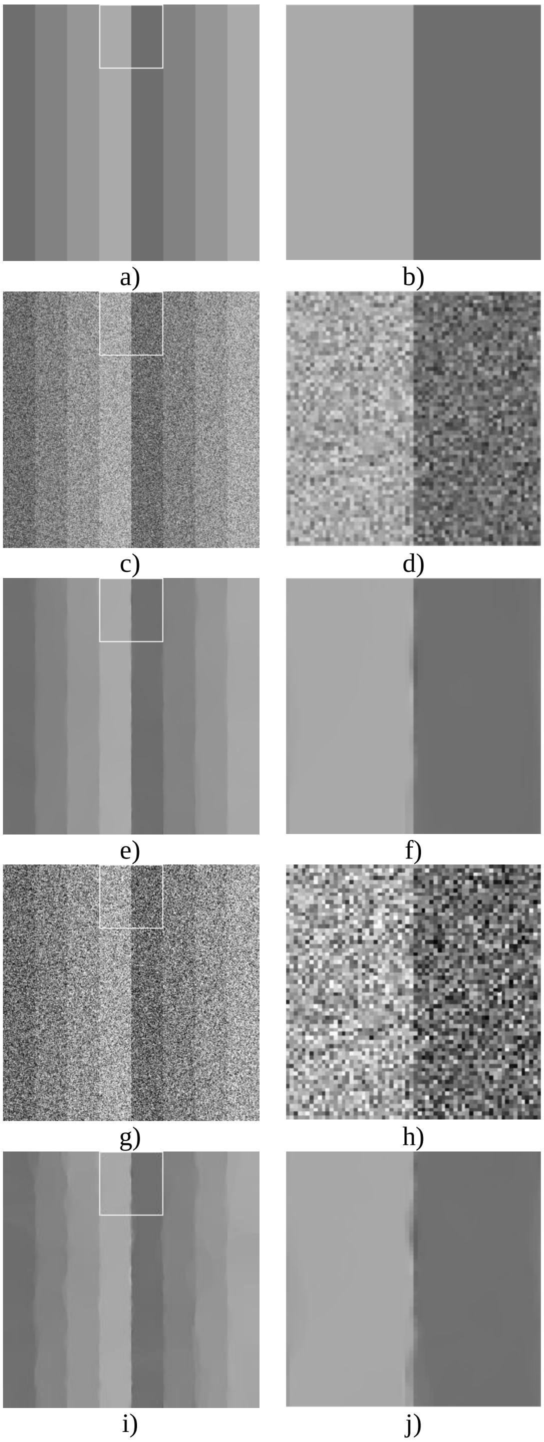

Figure 1: Denoising of the artificial image: a)-b) original image, c)-d) noisy image with linear combination of noises, e)-f) denoised image (c), g)-h) noisy image with superposition of noises, i)-j) denoised image (g).

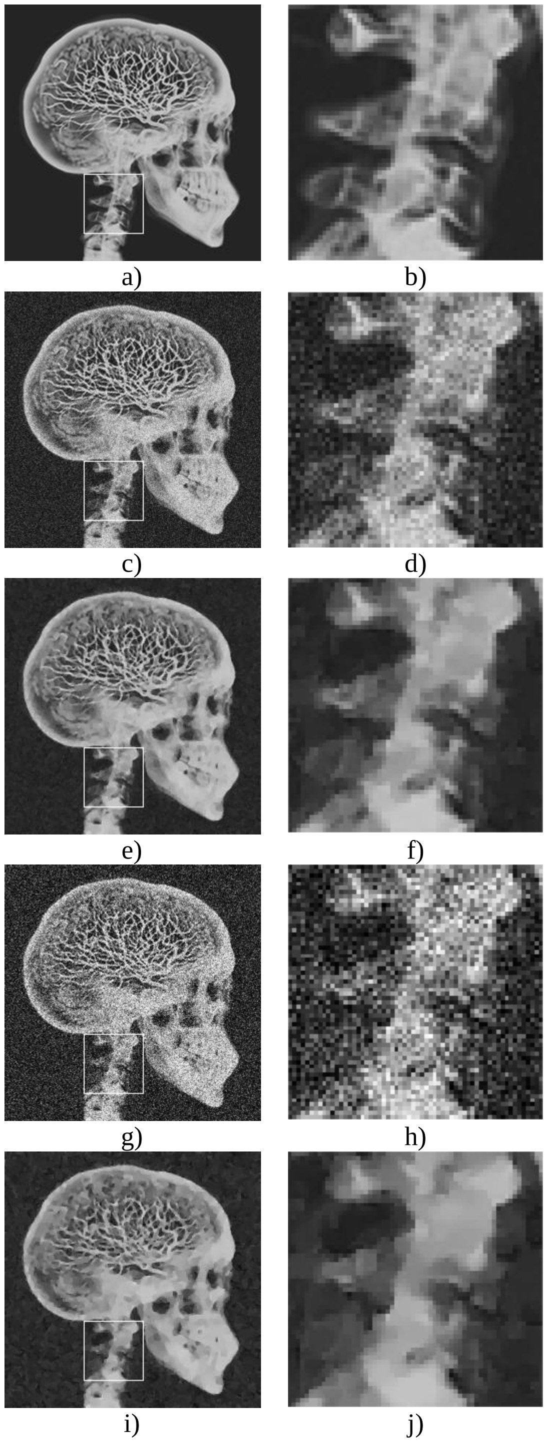

Figure 2: Denoising of the real image: a)-b) original image, c)-d) noisy image with linear combination of noises, e)-f) denoised image (c), g)-h) noisy image with superposition of noises, i)-j) denoised image (g).

interval (1.64%).

The final noisy image (linear combination of noises in Fig. 1c) is created with proportion 0.6 for Gaussian noisy image $v^{(1)}$ and proportion 0.4 for Poisson noisy image $v^{(2)}$: $v = 0.6v^{(1)} + 0.4v^{(2)}$.

Then we define proportion for linear combination as $\lambda_1 / \lambda_2 = (0.6 \times 47.1757)/(0.4 \times 11.7939) = 6/1$. Coefficients of linear combination are defined as $\lambda_1 = 6/7 = 0.8571$, $\lambda_2 = 1/7 = 0.1429$.

	Q_{PSNR}	Q_{SSIM}	Q_{MSE}
Noisy	19.4291	0.1073	741.5963
ROF	34.1236	0.8978	25.1606
Modified ROF	32.4315	0.8703	37.8791
PURE-LET	33.0309	0.9277	32.3587
Proposed method λ_1=0.8571, λ_2=0.1429, μ = 0.5003, σ = 47.1757	**41.1209**	**0.9841**	**4.9905**
Proposed method with automatically defined parameters λ_1=0.8414, λ_2=0.1586, μ = 0.5112, σ = 41.0314	41.0998	0.9840	5.0478

Table 1: Quality of noise removing for the artificial image with linear combination of noises.

	Q_{PSNR}	Q_{SSIM}	Q_{MSE}
Noisy	15.1406	0.0457	1990.8
ROF	31.4797	0.8364	21.2502
Modified ROF	28.4591	0.7871	27.5694
PURE-LET	28.9451	0.7986	25.9883
Proposed method λ_1=1, λ_2=0, μ = 0.3033, σ = 47.1757	**35.8011**	**0.9598**	**16.8122**
Proposed method with automatically defined parameters λ_1=0.9715, λ_2=0.0285, μ = 0.3021, σ = 46.0314	35.7589	0.9596	17.2658

Table 2: Quality of noise removing for the artificial image with Gaussian noise.

Values of Q_{PSNR}, Q_{MSE}, and Q_{SSIM} of the noisy image (linear combination of noises) are, respectively, 19.4291, 741.5963, and 0.1073.

In the case of the image with superposition of noises, we add Gaussian noise over Poisson noisy image. The intensity function of this Gaussian noisy image is $v^{(1)}$ too. As above, the grey values of $v^{(1)}$ need to be between 0 and 255. If the grey value of a pixel after adding of Gaussian noise is out of the interval from 0 to 255, it needs to be reset to $v_{ij}^{(1)} = v_{ij}^{(2)}$.

There are 1220 pixels out of this interval (1.86%). The noisy image (superposition of noises, Fig. 1g) is also Gaussian noisy image $v = v^{(1)}$. In this case, we don't know λ_1 and λ_2, therefore we use the algorithm with automatically defined parameters.

Values of Q_{PSNR}, Q_{MSE}, and Q_{SSIM} of the noisy image are, respectively, 14.9211, 2093.9827, and 0.0439.

	Q_{PSNR}	Q_{SSIM}	Q_{MSE}
Noisy	26.6776	0.3640	139.7396
ROF	36.4958	0.9381	14.5715
Modified ROF	**44.6347**	**0.9897**	**2.2001**
PURE-LET	37.4485	0.9404	10.5692
Proposed method λ_1=0, λ_2=1, μ = 0.8012, σ = 0.0001	44.6343	0.9897	2.2014
Proposed method with automatically defined parameters λ_1=0.0524, λ_2=0.9476, μ = 0.7923, σ = 2.0544	44.6156	0.9896	2.2466

Table 3: Quality of noise removing for the artificial image with Poisson noise.

	Q_{PSNR}	Q_{SSIM}	Q_{MSE}
Noisy	14.9211	0.0439	2093.983
ROF	31.2913	0.8346	48.3008
Modified ROF	30.5471	0.8232	56.5601
PURE-LET	33.9889	0.9298	25.9534
Proposed method with automatically defined parameters λ_1=0.8014, λ_2=0.1986, μ = 0.4812, σ = 40.0314	**37.3366**	**0.9677**	**12.0066**

Table 4: Quality of noise removing for the artificial image with superposition of noises.

Tables 1 – 4 show results for linear combination of noises, Gaussian noise, Poisson noise, and superposition of noises for the artificial image.

6.2 Real image with artificial noise

The artificial noise is generated by linear combination and superposition of Poisson and Gaussian noises.

For both cases, we consider Poisson noise first. Poisson noise variance is an average value $\bar{\sigma}_2 = 9.0882$. If the grey value of a pixel after adding of Poisson noise is out of the interval from 0 to 255, it needs to be reset to

$v_{ij}^{(2)} = u_{ij}$. Here there are no pixels out of this interval.

For Gaussian noise, we consider the variance of Gaussian noise is four times greater than the variance of Poisson noise $\sigma_1 = 4\bar{\sigma}_2 = 36.3529$. The real image is a human skull [14] (Fig. 2a). Others (Fig. 2b-j) show noisy and denoised images and zoomed out part of them.

For the case of linear combination of noises, we denote the intensity function of Gaussian noisy image as $v^{(1)}$. As above, the grey values of intensity function $v^{(1)}$ also need to be between 0 and 255. If the grey value of a pixel after adding of Gaussian noise is out of the interval from 0 to 255, it needs to be reset to $v_{ij}^{(1)} = u_{ij}$. In this case, there are 5355 pixels out of this interval (8.17%). The final image (linear combination of noises, Fig. 2c) is created with proportion 0.5 for Gaussian noisy image $v^{(1)}$ and proportion 0.5 for Poisson noisy image $v^{(2)}$: $v = 0.5v^{(1)} + 0.5v^{(2)}$. The proportion for linear combination is: $\lambda_1 / \lambda_2 = (0.5\times 36.3529)/(0.5\times 9.0882) = 4/1$.

Hence, coefficients of linear combination are defined as λ_1 = 4/5 =0.8, λ_2 = 1/5 = 0.2. Values of Q_{PSNR}, Q_{MSE}, and Q_{SSIM} of final noisy image are, respectively, 23.6878, 278.1619, and 0.5390.

For superposition of noises, we add Gaussian noise over Poisson noisy image. We denote the intensity function of Gaussian noisy image as $v^{(1)}$. As above, grey values of $v^{(1)}$ need to be between 0 and 255. If the grey value after adding of Gaussian noise is out of the interval from 0 to 255, it needs to be reset to $v_{ij}^{(1)} = v_{ij}^{(2)}$. In this case, there are 5621 pixels out of this interval (8.58%). The final noisy image (superposition of noises, Fig. 2g) is also the Gaussian noisy image $v = v^{(1)}$.

In this case, we don't know λ_1 and λ_2, therefore we use the algorithm to find them. Values of Q_{PSNR}, Q_{MSE}, and Q_{SSIM} of the final noisy image (superposition) are, respectively, 17.8071, 1077.3831, and 0.3242.

	Q_{PSNR}	Q_{SSIM}	Q_{MSE}
Noisy	23.6878	0.5390	278.1619
ROF	27.3974	0.8295	118.3975
Modified ROF	25.5644	0.7513	197.5403
PURE-LET	25.7781	0.8105	191.0341
Proposed method λ_1=0.8, λ_2=0.2, μ = 0.0524, σ = 36.3529	**27.6641**	**0.8331**	**110.9451**
Proposed method with automatically defined parameters λ_1=0.7804, λ_2=0.2196, μ = 0.0512, σ = 34.2311	27.6039	0.8325	112.8984

Table 5: Quality of noise removing for the real image with linear combination of noises.

	Q_{PSNR}	Q_{SSIM}	Q_{MSE}
Noisy	18.0693	0.3337	1014.3
ROF	24.0246	0.7299	257.4095
Modified ROF	23.2511	0.7019	311.8742
PURE-LET	23.8712	0.7989	265.6153
Proposed method λ_1=1, λ_2=0, μ = 0.0956, σ = 36.3529	**24.2011**	**0.8029**	**242.5101**
Proposed method with automatically defined parameters λ_1=0.9538, λ_2=0.0462, μ = 0.0902, σ = 35.0633	24.1882	0.8028	247.8894

Table 6: Quality of noise removing for the real image with Gaussian noise.

	Q_{PSNR}	Q_{SSIM}	Q_{MSE}
Noisy	28.4991	0.7625	91.8683
ROF	31.0567	0.9457	50.9818
Modified ROF	**31.1992**	**0.9022**	**48.9375**
PURE-LET	30.8955	0.8678	53.1066
Proposed method λ_1=0, λ_2=1, μ = 0.0541, σ = 0.0001	31.1334	0.8986	49.7922
Proposed method with automatically defined parameters λ_1=0.0491, λ_2=0.9509, μ = 0.0567, σ = 4.2012	31.1316	0.8986	50.1094

Table 7: Quality of noise removing for the real image with Poisson noise.

	Q_{PSNR}	Q_{SSIM}	Q_{MSE}
Noisy	17.8077	0.3242	1077.383
ROF	23.1936	0.7062	311.6856
Modified ROF	23.0413	0.7033	319.3831
PURE-LET	23.6278	0.7072	282.0349
Proposed method with automatically defined parameters λ_1=0.7704, λ_2=0.2296, μ = 0.1102, σ = 36.3412	**23.7292**	**0.7094**	**275.5229**

Table 8: Quality of noise removing for the real image with superposition of noises.

Tables 5 – 8 show results for linear combination of noises, Gaussian noise, Poisson noise, and superposition of noises for the real image.

6.3 About of initial solution

In order to create the initial image, we use the convolution operator. The table 9 shows the dependency of restored result for the initial image, where:

(a) Initial parameters $\lambda_1^0 = 0, \lambda_2^0 = 1, \mu = 1$;

(b) Initial parameters $\lambda_1^0 = \lambda_2^0 = 0.5, \mu = 1$;

(c) Initial solution u^0 is given as a randomized matrix;

(d) Initial solution $u^0 = \Lambda * v$ is given as an average value of neighbour pixels by the convolution operator with the mask $\Lambda = (1/9)$ of the size 3x3.

Table 9 shows the best result of denoising is (d) by criteria PSNR and MSE.

The result (c) by SSIM looks different in contract to ones in Tables 1-8. It illustrates incorrectness of a randomized initial solution (accidental and not stable, if a probability distribution is unknown).

Next, we have to notice that the non-optimal result (a) has been used in experiments for Table 5. It appears to be enough for the good result with automatically defined model parameters.

	(a)	(b)	(c)	(d)
λ_1	0.7804	0.8094	0.8733	0.8032
λ_2	0.2196	0.1906	0.1267	0.1968
μ	0.0512	0.0573	0.0653	0.0565
σ	34.2311			
Q_{PSNR}	27.6039	27.2214	26.5611	**27.6523**
Q_{MSE}	112.8984	120.4355	132.0264	**107.5431**
Q_{SSIM}	0.8325	0.8317	**0.8395**	0.8392

Table 9: Dependency of denoising on initial solution.

At last, the variant (b) initially looks better than (a) for kind of better assumption of $\lambda_1^0 = \lambda_2^0 = 0.5$ to process the real image. Nevertheless, our assumption about $\mu = 1$ is very far from the good one, and evidently the limit of the number of steps $K = 500$ is insufficient in this case.

As a result, the variant (d) is the best idea for initial solution.

7 Conclusion

In this paper, we proposed a novel method that can effectively remove the mixed Poisson-Gaussian noise. Furthermore, our proposed method can be also used to remove Gaussian or Poisson noise separately. This method is based on the variational approach.

The denoising result strongly depends on values of coefficients of linear combination λ_1 and λ_2. These values can be set manually or can be defined automatically. When processing real images, we can use the proposed method with automatically defined parameters.

Although our method concentrates on removing the linear combination of noise, but it also efficiently removes the superposition of noises. In this case, we consider the superposition of noises is equivalent to some linear combination of them with coefficients found in iteration process.

In this paper we show that our simple model “feels” well the wide range of proportion of two types of noises. As a result, it appears to be the good basis for removing superposition of such noises.

It is evident, the iteration process (6) used here is insufficiently effective in comparing with other possible computational schemes. In this paper, we try to compare our approach to image denoising with PURE-LET method only in possible reduction of our model complexity, not in others.

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